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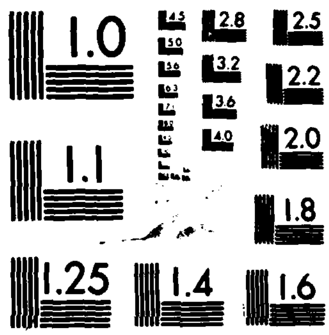

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The Air Force Office of Scientific Research (AFOSR) contract is concerned with experimental and theoretical investigations on electrically conducting aerosols for electromagnetic obscuration applications. Presented below is the progress made in the theoretical analysis of aerosols consisting of magnetic (Fe,Co,Ni) particle chains in air. Particular attention is given to the effects of an external magnetic field on the chain equilibrium and the coagulation dynamics.

1. A theory of the *diffusion of macroscopic, magnetic particles* (suspended in gaseous or liquid media) in density and magnetic field gradients has been developed from first principles (Fokker-Planck equation), in which the influence of the random, fluctuating magnetic fields, produced collectively by the magnetic dipole particles in their thermal motions on the diffusing particle, is taken into account in a self-consistent way. As an application, the steady-state boundary-value problem for the diffusion of magnetic grains in the inhomogeneous magnetic field of an adsorbing sink dipole and an external, homogeneous magnetic field has been solved by means of a stream and Green's functions approach. The coagulation coefficient for magnetic dipole particles in the presence of an external magnetic field have been derived and applied to the coagulation of magnetic grains and the formulation of magnetic chains in magnetic aerosols with an external magnetic field.

2. Idealized statistical calculations of chaining in a dilute suspension of macroscopic magnetic particles in a rarefied gas have been made in the presence of an external, homogeneous magnetic field. The primary colloidal particles are assumed to be spherical, of the same size, and have saturated magnetic moments. The magneto-chemical potentials and the association-dissociation equations have been derived for chains consisting of $\nu \geq 1$ magnetic grains as a function of the temperature T, the density N_ν of chains, and the homogeneous magnetic field $\vec{B}_0$. High field intensities B_0 are shown to shift the chain length distribution $F = F(\nu)$ in favor of long chains, $\nu \gg 1$, whereas increasing temperatures T move the maximum of this statistical distribution to smaller chain lengths, $\nu \rightarrow 1$. The theory appears to be in qualitative agreement with experiments using an external magnetic field for the alignment of chains and their stabilization.

3. The artificial charging of magnetic aerosols is of interest with regard to the reduction of the coagulation rate of Coulomb repulsion of the magnetic particles. For this reason, the interesting properties of charged particle gases and their fluctuating electric microfields were studied. It is shown that (i) the collective electric fields act at distances larger than the characteristic repulsion distance $D = (KT/4\pi ne^2)^{1/2}$ of like-charged particles and (ii) the average collective electric field is $E_W = (12\pi nKT)^{1/2}$ for ideal gases of particles of the same charge e. Thus, in a thermal equilibrium gas of like-charged particles, the longitudinal microfields are considerably stronger than in a plasma, since in the latter the random electric fields of the negative and positve charges nearly

compensate each other. In addition, the interrelation between (average) kinetic, interaction, collective field, and electric self-energies has been calculated for charged particle gases.

4. Finally, an investigation on the reformulation of electromagnetic theory for space applications was carried through in which the existence of (i) a preferred frame of reference and (ii) an electromagnetic carrier for the electromagnetic waves is taken explicitly into account. In view of the experimental discovery of the cosmic microwave background, (i) a preferred frame of reference and (ii) an electromagnetic substratum can no longer be denied. The latter work was supported only in part by the AFOSR.

A list of four papers follows. Two of them have been accepted for publication, while the others are still under review. We ask the AFOSR to accept reprints of these publications as a final report. These papers are attached as Appendices A through D, respectively.

A. H. E. Wilhelm, "Dissociation-Association Equilibrium of Magnetic Particle Chains in Homogeneous Magnetic Fields;" Phys. Fluids (1985).

B. H. E. Wilhelm, "Diffusion and Coagulation of Magnetic Dipole Particles in Inhomogeneous Magnetic Fields," Phys. Fluids (1985).

C. H. E. Wilhelm, "Statistical Distribution of Collective Electric Fields in Charged Particle Gases;" Int. J. Electron. (1985) - to be published.

D. H. E. Wilhelm, "Covariant Electromagnetic Theory for Inertial Frames with Substratum Flow," Radio Sci. (1985) - to be published.

APPENDIX A

DISSOCIATION-ASSOCIATION EQUILIBRIUM OF MAGNETIC PARTICLE CHAINS IN HOMOGENEOUS MAGNETIC FIELDS

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ABSRACT

An idealized statistical theory of chaining in a dilute suspension of macroscopic magnetic particles in a rarefied gas is presented when an external homogeneous magnetic field is present. The primary colloidal particles are assumed to be spherical, of the same size, and to have saturated magnetic moments. The magneto-chemical potentials and the association-dissociation equations are derived for chains consisting of $v \geq 1$ magnetic grains, in dependence of the temperature T, the density N_v of chains, and the homogeneous magnetic field $\mathbf{B}_0$. High field intensities B_0 are shown to shift the chain length distribution $F = F(v)$ in favor of long chains, $v \gg 1$, whereas increasing temperatures T move the maximum of this statistical distribution to smaller chain lengths, $v \rightarrow 1$. The theory appears to be in qualitative agreement with oven experiments using an external magnetic field for the alignment of the chains and their stabilization.

I. INTRODUCTION

It is known that external electric or magnetic fields shift the molecular dissociation equilibrium in favor of the paraelectric, respectively, paramagnetic reaction products.[1] Statistical considerations show that a homogeneous magnetic field $\mathbf{B}_0$ affects the thermal ionization equilibrium $a \rightleftarrows i + e$ of a plasma if the quanta of the oscillatory electron (e) motion in the $\mathbf{B}_0$ field that corresponds classically to the electron gyration are of the order or larger than the thermal energy, $\hbar\omega \geq KT$, where $\omega = eB_0/m$ (e is the charge and m is the mass of an electron).[2] The ionization may be enhanced or depressed depending on whether the atoms (a) are diamagnetic or paramagnetic.[2] The electron (e_-) and positron (e_+) densities of the thermal vacuum equilibrium $e_+ + e_- \rightleftarrows 2\gamma$ (gamma quanta) are increased by a homogeneous magnetic field $\mathbf{B}_0$ (in favor of the paramagnetic or spin particles $e_\pm$).[3] Similarly, the electron-hole equilibria in solids are shifted towards higher electron and hole concentrations by a homogeneous magnetic field, in particular in crystal structures with small effective electron and hole masses.[4] However, these interesting physical effects are in general quantitatively not very significant since the energy pB_0 of the magnetic moments p in the magnetic field is small compared with the thermal energy, $pB_0 \ll KT$, except in the case of suprathermal fields $B_0 \geq KT/p$, which can be generated through magnetic flux compression.[5,6]

Magnetic field effects on reaction equilibria are quantitatively extremely important if the reacting species are macroscopic or colloidal particles that have paramagnetic or ferromagnetic moments $p \gg p_B$, which are large compared with the Bohr magneton $p_B = e\hbar/2m = 9.274 \times 10^{-24}\ \mathrm{Am}^2$. (An analopeous conclusion holds for paraelectric or ferroelectric macroparticles in electric fields.) For the latter reason, the magnetic dipole energy pB_0 may already be larger than the thermal energy, $pB_0 > KT$, for moderate magnetic field strengths B_0, so that the reaction equilibrium is significantly affected by the magnetic field. In particular, this is true for colloidal suspensions of saturated paramagnetic or ferromagnetic grains of radius $a \sim 10^{-6}$ m, which associate to long chains consisting of $\nu \gg 1$ grains in the presence of an external, homogeneous magnetic field. Dispersons of such colloidal chains of electrically conducting, ferromagnetic particles (Fe, Co, Ni) in the atmosphere are of technical interest as wide-band electromagnetic obscurants.

Herein, we analyze the association-dissociation equilibrium of chains a_ν consisting of ν spherical grains of radius $r = a$ and saturated magnetic moment $\mathbf{p}_1$ dispersed in dilute,

homogeneous carrier gases of temperature T and a homogeneous external magnetic field $\mathbf{B}_0$.
The corresponding reaction equations are

$$a_\nu \rightleftarrows a_{\nu-1} + a_1 \ , \quad 2 \leq \nu < \nu_\infty \ , \tag{1}$$

where $a_1 \equiv a$ and $\nu_\infty < \infty$ is the maximum number of grains in a chain. If the grain material is treated as a single, magnetic domain of magnetization M[A/m], the magnetic moment [Am^2] of a spherical grain of radius $r = a$ is given by

$$\mathbf{p} = 4\pi(\pi/6)a^3\mathbf{M} \ . \tag{2}$$

In local magnetic field $\mathbf{B}_0$ the potential energy of a grain is $U = -\mathbf{p} \cdot \mathbf{B}_0 \gtrless 0$. The average moment $\langle p_\parallel \rangle$ of a grain in direction of field $\mathbf{B}_0$ is in thermal equilibrium $\langle p_\parallel \rangle = L(pB_0/KT)$ where $L(\varepsilon_0) = \coth\varepsilon_0 - \varepsilon_0^{-1} \simeq 1$, $\varepsilon_0 \gg 1$, is the Langevin function. Accordingly, $\langle p_\parallel \rangle = p$ for $pB_0 \gg KT$. The maximum potential energy of the dipoles in the magnetic field $\mathbf{B}_0$ (relative to the thermal energy KT) is by Eq. (2)

$$\varepsilon_o = 4\pi(\pi/6)a^3MB/KT = 4.77 \times 10^{23}\, a^3MB/T \ . \tag{3}$$

Accordingly, $\varepsilon_0 \sim 10^2$ for $M = 10^6/4\pi$ A/m, $B_0 = 1$ Vsec/m^2, $T = 300°K$, and $a = 10^{-6}$ m. This example shows that at standard temperatures T, the grains have their moment $\mathbf{p}$ aligned parallel to $\mathbf{B}_0$ already at moderate field intensities, $\langle p_\parallel \rangle = \mathbf{p} \parallel \mathbf{B}_0$ for $\varepsilon \gg 1$.

Since the magnetic field of a dipole $\mathbf{p}_i$ is $\mathbf{B}_i = -\nabla(\mu_0\mathbf{p}_i \cdot \mathbf{r}/4\pi r^3)$, the interaction energy of two magnetic dipoles "i" amd "j" at a distance $r = |\mathbf{r}|$ apart is $U_{ij} = (\mu_0/4\pi)r^{-3}[\mathbf{p}_i \cdot \mathbf{p}_j - 3r^{-2}(\mathbf{p}_i \cdot \mathbf{r})(\mathbf{p}_j \cdot \mathbf{r})]$ where $\mu_0 = 4\pi \times 10^{-7}$ Vsec/Am. The average distance between grains is on the order $\bar{r} \sim n^{-1/3}$ where $n[m^{-3}]$ designates their density. Accordingly, the average binary interaction energy of the dipoles is

$$\bar{\varepsilon}_{ij} = 4\pi\mu_o(\pi/6)^2a^6M^2n/KT = 3.14 \times 10^{17}\, a^6M^2n/T \tag{4}$$

relative to the thermal energy KT. Hence, $\bar{\varepsilon}_{ij} = 6.63 \times 10^{-3} \ll 1$ for $M = 10^6/4\pi$ A/m, $T = 300°K$, $a = 10^{-6}$ m, and $n \leq 10^{21}$ m^{-3}. Thus, the grains behave like an ideal gas at $T = 300°K$ for densities $n < 10^{21}$ m^{-3}. In the following it will be assumed that the grains and the chains formed from them behave like a quasi-ideal suspension in the atomic (nonmagnetic) carrier gas. By Eq. (4) this requires $\bar{\varepsilon}_{ij} \ll 1$, i.e., sufficiently low particle densities n and sufficiently high system temperatures T (below the Curie point).

Physically more complex is the calculation of the chain lengths equilibrium for ferromagnetic grains dispersed at highest densities ($n \sim 10^{23}\ m^{-3}$) in so-called ferromagnetic fluids.[7] For this situation, an asymptotic theory for strongly nonideal interactions was proposed (which breaks down in the ideal limit).[7] This nonideal theory has been shown to disagree up to orders of magnitude with the experimental data.[8]

II. MAGNETOACTIVE CHAIN EQUILIBRIUM

A homogeneous magnetic field $\mathbf{B}_0$ aligns the magnetic dipole moments $\mathbf{p}_1$ of the grains a_1 and thus provides favorable conditions for the formation of linear particle chains resembling rather rigid one-dimensional polymers. In thermal equilibrium, the chain lengths L_v or the numbers $v = 1, 2, 3, \ldots, v_\infty$ of grains a_1 in the v-chains have a distribution $F = F(v)$ determined by the maximum entropy principle.[9] The most elementary dissociation ($\rightarrow$) –association ($\leftarrow$) reactions are those in which one grain a_1 is removed or added to a v-chain from the collective $2 \leq v < v_\infty$:

$$\begin{aligned} a_v &\leftrightarrows a_{v-1} + a_1 \ , \\ a_v &\leftrightarrows a_{v-2} + a_1 \ , \\ &- - - - - \\ a_3 &\leftrightarrows a_2 + a_1 \ , \\ a_2 &\leftrightarrows a_1 + a_1 \ . \end{aligned} \tag{5}$$

Adding these relations yields a summary reactin equation, which describes the dissociation of a v-chain into v grains a_1 and the formation of a v-chain by association of v grains a_1, respectively:

$$a_v \rightleftharpoons v a_1 \quad , v = 2, 3, 4, \ldots, v_\infty \ . \tag{6}$$

This reaction is of particular interest since it relates the densities N_v of the chains $v \geq 2$ to the density N_1 of grains a_1. The distribution N_v of the v-chains observed in experiments is the one with largest probability and, hence, maximizes the entropy of the colloidal suspension. The latter condition leads to a statistical equilibrium equation for the summary reaction (6) in terms of the magneto-chemical potential Ω_v of the v-chains and the magneto-chemical potential Ω_1 of the grains a_1,

$$\Omega_v = v\Omega_1 \ , \quad v = 2, 3, 4, \ldots, v_\infty \ . \tag{7}$$

The magneto-chemical potentials of the chains $v > 1$ and the grains $v = 1$ are given for ideal conditions by

$$-\Omega_\nu = KT\{ln(N_\nu\Lambda_\nu^3) + \widehat{\varepsilon}_\nu/KT - ln(U_\nu^{VR}U_\nu^B)\} \tag{8}$$

where

$$\Lambda_\nu = h/(2\pi m_\nu KT)^{1/2} \ , \tag{9}$$

$$U_\nu^{VR} = U_\nu^V U_\nu^R \ , \tag{10}$$

$$U_\nu^B = sinh(p_\nu B_o/KT)/(p_\nu B_o/KT) \ , \tag{11}$$

are the thermal DeBroglie wavelength of the ν-chains of mass m_ν, the product of their vibrational (V) and rotational (R) partition functions, and the orientational partition functions of the magnetic moments $\mathbf{p}_\nu$ of the ν-chains. Obviously,

$$U_\nu^B = \int_0^{2\pi} d\phi \int_0^{\pi} exp(p_\nu B_o \cos\theta) \sin\theta \, d\theta \ , \tag{12}$$

where $\phi = \sphericalangle(\mathbf{p}_\nu, \mathbf{B}_0)$. The masses m_ν and moments p_ν are given in terms of m_1 and p_1, respectively,

$$m_\nu = \nu m_1 \ , \ p_\nu = \nu p_1 \ . \tag{13}$$

Substitution of Eq. (8) into Eq. (7) yields the fundamental dissociation-association equations for ν-chains in a homogeneous magnetic field $\mathbf{B}_0$:

$$N_\nu = N_1^\nu \nu^{3/2} f_\nu^{VR} \frac{p_1}{p_\nu} \frac{sinh(p_\nu B_o/KT)}{[sinh(p_1 B_o/KT)]^\nu} \left[\frac{p_1 B_o}{KT} \frac{h^3}{(2\pi m_1 KT)^{3/2}} \right]^{\nu-1} exp\left(\frac{\Delta\widehat{\varepsilon}_\nu}{KT}\right),$$

$$\nu = 2, 3, 4 \ldots, \nu_\infty \tag{14}$$

where

$$\Delta\widehat{\varepsilon}_\nu \equiv \nu\widehat{\varepsilon}_1 - \widehat{\varepsilon}_\nu \sim \nu\{\mu_o p_1^2/4\pi(2a_1)^3\} \ , \tag{15}$$

$$f_\nu^{VR} \equiv U_\nu^{VR}/(U_1^{VR})^\nu \sim 10^0 \ . \tag{16}$$

The energy $\Delta\widehat{\varepsilon}_\nu$ required for the dissociation of a ν-chain into ν grains is of the magnitude $\Delta\widehat{\varepsilon}_\nu \sim \nu 10^{-1}$ eV. The values of $\Delta\widehat{\varepsilon}_\nu$ can be calculated (ν-body problem) or measured experimentally. Equation (16) holds in view of the macroscopic nature of the particles $\nu \geq 1$.

Equation (14) represents $\nu_\infty - 1$ equations for the determination of the ν_∞ unknown particle densities $N_1, N_2, N_3, \ldots, N_{\nu_\infty}$. A complete system of equations is obtained by adding the equation for the conservation of the total grain density N,

$$\sum_{\nu=1}^{\nu_\infty} \nu N_\nu = N \tag{17}$$

Substitution of Eq. (14) shows that Eq. (17) represents a polynomial of order ν_∞ for the determination of the grain density N_1,

$$\sum_{\nu=1}^{\nu_\infty} C_\nu(B_o,T) N_1^\nu = N \tag{18}$$

where

$$C_\nu(B_o,T) = \nu^{3/2} f_\nu^{VR} \frac{p_1}{p_\nu} \frac{\sinh(p_\nu B_o/KT)}{[\sinh(p_1 B_o/KT)]^\nu} \left[\frac{p_1 B_o}{KT} \frac{h^3}{(2\pi m_1 KT)^{3/2}} \right]^{\nu-1} \exp\left(\frac{\Delta\widehat{\varepsilon}_\nu}{KT} \right) ,$$

$$\nu = 2,3,4,\ldots,\nu_\infty \; ; C_1(B_o,T) \equiv 1 \; . \tag{19}$$

With N_1 calculated as the positive root of Eq. (18), the chain densities follow from Eq. (14) as $N_\nu = C_\nu(B_0,T) N_1^{\,\nu}$.

In applications, one is mainly concerned with suprathermal magnetic fields for which

$$B_o \gg KT/p_1 \; : \; p_\nu B_o/KT \gg 1 \; . \tag{20}$$

For such strong magnetic fields, Eq. (19) reduces to the dissociation-association equation

$$N_\nu = N_1^\nu 2^{\nu-1} \nu^{1/2} f_\nu^{VR} \left[\frac{p_1 B_o}{KT} \frac{h^3}{(2\pi m_1 KT)^{3/2}} \right]^{\nu-1} \exp\left(\frac{\Delta\widehat{\varepsilon}_\nu}{KT} \right) ,$$

$$\nu = 2,3,4,\ldots,\nu_\infty \; ; B_o \gg KT/p_1 \; , \tag{21}$$

with

$$C_\nu(B_o, T) = 2^{\nu-1}\nu^{1/2} f_\nu^{VR} \left| \frac{p_1 B_o}{KT} \frac{h^3}{(2\pi m_1 KT)^{3/2}} \right|^{\nu-1} exp\left(\frac{\Delta\hat{\varepsilon}_\nu}{KT}\right),$$

$$\nu = 2,3,4,\ldots,\nu_\infty \; ; B_o \gg KT/p_1 \quad , \qquad (22)$$

Equation (21) provides interesting proportional relations that exhibit the main magnetic field (B_0) and temperature (T) dependences of the chain densities N_ν:

$$N_2 \propto N_1^2 B_o^1 T^{-5/2} \, exp(\Delta\hat{\varepsilon}_2/KT)$$

$$N_3 \propto N_1^3 B_o^2 T^{-10/2} \, exp(\Delta\hat{\varepsilon}_3/KT)$$

$$N_4 \propto N_1^4 B_o^3 T^{-15/2} \, exp(\Delta\hat{\varepsilon}_4/KT) \, , \quad B_o \gg KT/p_1 . \qquad (23)$$

$$- - - - - - - - - -$$

$$N_\nu \propto N_1^\nu B_o^{\nu-1} T^{-5(\nu-1)/2} \, exp(\Delta\hat{\varepsilon}_\nu/KT)$$

It is seen that an increase of the magnetic field B_0 shifts the distribution $\{N_1, N_2, \ldots, N_{\nu_\infty}\}$ in favor of the large chains, $\nu \to \nu_\infty$, whereas an increase in the temperature T shifts the distribution $\{N_\nu\}$ in favor of the short chains, $\nu \to 1$. In view of the exponential T-dependence in Eq. (23) and $\Delta\varepsilon_\nu > \Delta\varepsilon_{\nu-1}$, a temperature increase has a particularly strong destructive effect on the long chains.

III. CONCLUSIONS

The statistical theory presented should be considered a first step towards the qualitative understanding of the dissociation-association equilibrium of magnetic colloid chains in an external, homogeneous magnetic field, presumed that the primary grains formed originally a sufficiently dilute suspension in a nonmagnetic gas. The favorable effect of the external magnetic field on the chain formation and the shifting of the chain lengths distribution $F(\nu)$ towards larger chain lengths ν has been observed experimentally. However, quantitative experimental data on magnetic chain lengths distributions in aerosols, which could be used for comparison with the theory, have apparently not yet been published. The experiments also indicate that chains and grains coagulate to large clusters which "fall out" in the gravitational field when the magnetic field is switched off. The rapid coagulation in the absense of a (sufficiently strong) magnetic field can be reduced by spraying the macroscopic particles with electric charges (Coulomb repulsion).[10]

In order to achieve a quantitative understanding of the chain equilibrium in an external magnetic field, several (difficult) problems remain to be solved rigorously. The dissociation energies $\Delta\widehat{\varepsilon}_\nu = \nu\widehat{\varepsilon}_1 - \widehat{\varepsilon}_\nu$ are to be calculated from the ν-body problem of a ν-chain made up of extended magnetic dipoles (grains of radius $a > 0$). Based on this ν-body dynamics, the oscillatory and rotational partition functions of the ν-chains have to be determined (even though these degrees of freedom of macroscopic particles are only poorly excited at temperatures below the Curie point). Finally, to render the theory applicable to higher grain densities, the nonideal interactions between chains and grains would have to be taken into consideration in the evaluation of the statistical chain equilibrium.

In connection with the chaining phenomenon, various other interesting effects could be investigated theoretically. For example, it would be important to understand the effect of primary grains that have not the same size but a size distribution. Furthermore, it would be interesting to evaluate the chain lengths distribution for primary grains that carry an artificial electric charge. Already these few examples indicate that the research on magnetically active colloids offer significant opportunities for further contributions.[10]

APPENDIX A: Alternative Derivation

It is instructive to derive the dissociation-association equation [Eq. (14)] also from the reaction

$$a_\nu \leftrightarrows a_{\nu-1} + a_1 \ , \quad \nu = 2, 3, 4, \ldots, \nu_\infty \ , \tag{A-1}$$

where

$$\Omega_\nu = \Omega_{\nu-1} + \Omega_1 \ , \quad \nu = 2, 3, 4, \ldots, \nu_\infty \ , \tag{A-2}$$

in statistical equilibrium. Substitution of the magneto-chemical potentials [Eq. (8)] into Eq. (A-2) yields (after some alegebra) the "one-grain" dissociation-association equation:

$$N_\nu = D_\nu(B_o, T) N_{\nu-1} N_1 \ , \quad \nu = 2, 3, 4, \ldots, \nu_\infty \ , \tag{A-3}$$

where

$$D_\nu(B_o, T) = g_\nu^{VR} \left(\frac{\nu}{\nu-1} \right)^{3/2} \frac{p_{\nu-1}}{p_\nu} \frac{sinh(p_\nu B_o/KT)}{sinh(p_{\nu-1} B_o/KT) sinh(p_1 B_o/KT)}$$

$$\times \left[\frac{p_1 B_o}{KT} \frac{h^3}{(2\pi m_1 KT)^{3/2}} \right] exp\left[(\hat{\varepsilon}_{\nu-1} + \hat{\varepsilon}_1 - \hat{\varepsilon}_\nu)/KT \right] , \quad \nu = 2, 3, 4 \ldots, \nu_\infty \ , \tag{A-4}$$

and

$$g_\nu^{VR} = U_\nu^{VR} / U_{\nu-1}^{VR} U_1^{VR} \ . \tag{A-5}$$

Eq. (A-3) represents a recurrent relation that gives, by elimination, the "ν-grain" dissociation-association equation:

$$N_\nu = N_1^\nu \prod_{\nu=2}^{\nu} D_\nu(B_o, T) \ , \quad \nu = 2, 3, 4 \ldots, \nu_\infty \ , \tag{A-6}$$

where

$$\prod_{\nu=2}^{\nu} D_\nu(B_o, T) = G_\nu [sinh(p_1 B_o/KT)]^{-(\nu-1)} \left[\frac{p_1 B_o}{KT} \frac{h^3}{(2\pi m_1 KT)^{3/2}} \right]^{\nu-1} exp\left(\frac{\nu \hat{\varepsilon}_1 - \hat{\varepsilon}_\nu}{KT} \right) \tag{A-7}$$

and

$$G_\nu = \prod_{\nu=2}^{\nu} g_\nu \times \left(\frac{\nu}{\nu-1} \right)^{3/2} \times \frac{p_{\nu-1}}{p_\nu} \times \frac{sinh(p_\nu B_o/KT)}{sinh(p_{\nu-1} B_o/KT)} \tag{A-8}$$

since

$$\sum_{\mathrm{v}=2}^{\mathrm{v}} (\hat{\varepsilon}_{\mathrm{v}-1} + \hat{\varepsilon}_1 - \hat{\varepsilon}_{\mathrm{v}}) = \mathrm{v}\hat{\varepsilon}_1 - \hat{\varepsilon}_{\mathrm{v}} \ . \tag{A-9}$$

Factorization of the finite product (A-8) and evaluation of the individual products gives

$$G_{\mathrm{v}} = f_{\mathrm{v}} \times \mathrm{v}^{3/2} \times \frac{p_1}{p_{\mathrm{v}}} \times \frac{sinh(p_{\mathrm{v}} B_o / KT)}{sinh(p_1 B_o / KT)} \tag{A-10}$$

since

$$\prod_{\mathrm{v}=2}^{\mathrm{v}} g_{\mathrm{v}} = f_{\mathrm{v}} \tag{A-11}$$

by Eqs. (A-5) and (16). Combining Eqs. (A-6), (A-7), and (A-9) results in the dissociation-association equation:

$$N_{\mathrm{v}} = N_1^{\mathrm{v}} f_{\mathrm{v}}^{VR} \mathrm{v}^{3/2} \frac{p_1}{p_{\mathrm{v}}} \frac{sinh(p_{\mathrm{v}} B_o / KT)}{[sinh(p_1 B_o / KT)]^{\mathrm{v}}} \left[\frac{p_1 B_o}{KT} \frac{h^3}{(2\pi m_1 KT)^{3/2}} \right]^{\mathrm{v}-1} exp\left(\frac{\mathrm{v}\hat{\varepsilon}_1 - \hat{\varepsilon}_{\mathrm{v}}}{KT}\right),$$

$$\mathrm{v} = 2, 3, 4 \ldots \mathrm{v}_{\infty} \ . \tag{A-12}$$

Equation (A-12) is identical to Eq. (14) derived from the summary reaction [Eq. (6)] and the statistical equilibrium relation (7) . The derivation of Eq. (14) is apparently simpler than that of Eq. (A-12). The identity is necessary for physical reasons.

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APPENDIX B

DIFFUSION AND COAGULATION OF MAGNETIC DIPOLE PARTICLES IN INHOMOGENEOUS MAGNETIC FIELDS

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ABSTRACT

A theory of the diffusion of macroscopic, magnetic particles (suspended in gaseous or liquid media) in density and magnetic field gradients is developed from first principles (Fokker-Planck equation). The influence of the random, fluctuating magnetic fields, produced collectively by the magnetic dipole particles in their thermal motions on the diffusing particle, is taken into account in a self-consistent way. It is shown that the anisotropy in the particle diffusion, caused by the coupling of translational and rotational degrees of freedom (Magnus effect), is small in most physical situations. As an application, the steady-state boundary-value problem for the diffusion of magnetic grains in the inhomogeneous magnetic field of an adsorbing sink dipole and an external, homogeneous magnetic field is solved by means of a stream and Green's functions approach. The coagulation coefficient for magnetic dipole particles in the presence of an external magnetic field is derived. The results are discussed with regard to the coagulation of magnetic grains and the formation of magnetic chains.

INTRODUCTION

The physical behavior of colloids formed from ferromagnetic grains suspended in magnetically inactive liquids has recently been investigated experimentally[1,2] and theoretically.[3,4] Under the influence of a uniform external magnetic field, the magnetic grains tend to associate in the form of long chains if the grain density and the temperature of the liquid are sufficiently low.[5,6]

Based on the Fokker-Planck equation, we develop an analytical theory of the thermal diffusion of magnetic dipole particles in magnetically passive fluids F (gases or liquids) when an inhomogeneous magnetic field $\vec{B}(\vec{r},t)$ is present. A novel diffusion equation is derived and applied to the diffusion of magnetic grains in the inhomogeneous field $\vec{B}_s(\vec{r})$ of an "absorbing" magnetic dipole when a homogeneous external field $\vec{B}_0$ is present. The corresponding boundary-value problem is solved analytically. The coagulation of dipole particles, which interact through their magnetic self-field, is discussed in dependence of the external field $\vec{B}_0$.

Ferromagnetic grains have a typical radius $a \sim 10^{-8}$ m. If the grain material is treated as a single magnetic domain of magnetization M[A/m], the magnetic moment [Am^2] of a spherical grain of radius r = a is given by

$$\vec{p} = 4\pi(\pi/6)a^3\vec{M} \quad . \tag{1}$$

Let these grains be dispersed in a nonmagnetic (μ_0) carrier medium (F) of temperature T. In a local magnetic field $\vec{B}$, the potential energy of a grain is

$$U = -\vec{p}\cdot\vec{B} \gtrless 0 \quad . \tag{2}$$

The average moment $\langle p_\parallel \rangle$ of a grain in direction of field $\vec{B}$ is in thermal equilibrium

$$\langle p_\parallel \rangle = L(\varepsilon)p, \quad \varepsilon \equiv pB/KT \quad , \tag{3}$$

where

$$L(\varepsilon) = \coth\varepsilon - \varepsilon^{-1} \cong 1, \quad \varepsilon \gg 1 \tag{4}$$

is the Langevin function. Accordingly, $\langle p_\parallel \rangle = p$ for $pB \gg KT$. The maximum potential energy of the dipoles in field $\vec{B}$ (relative to the thermal energy KT) is by Eq. (1)

$$\varepsilon = 4\pi(\pi/6)a^3MB/KT = 4.77 \times 10^{23}\, a^3MB/T \quad . \tag{5}$$

Accordingly, $\varepsilon \sim 10^2$ for $M = 10^6/4\pi$ A/m, $B = 1$ Vsec/m^2, $T = 300$°K, and $a = 10^{-8}$ m. This example shows that at standard temperatures T, the grains have their moment $\vec{p}$ aligned parallel to $\vec{B}$ already at moderate intensities, $\langle \vec{p}_\parallel \rangle = \vec{p} \parallel \vec{B}$ for $\varepsilon \gg 1$.

Since the magnetic field of a dipole $\vec{p_i}$ is $\vec{B_i} = -\nabla(\mu_o\vec{p_i}\cdot\vec{r}/4\pi r^3)$, the interaction energy of two dipoles "i" and "j" at a distance $r = |\vec{r}|$ apart is ($\mu_o = 4\pi \times 10^{-7}$ Vsec/Am)

$$U_{ij} = (\mu_o/4\pi)r^{-3}[\vec{p}_i\cdot\vec{p}_j - 3r^{-2}(\vec{p}_i\cdot\vec{r})(\vec{p}_j\cdot\vec{r})] \quad . \tag{6}$$

The average distance between the dipoles is of the order $\bar{r} \sim n^{-1/3}$ where $n[m^{-3}]$ designates their density. The average binary interaction energy of the dipoles is by Eq. (6) of the order

$$\varepsilon_{ij} = 4\pi\mu_o(\pi/6)^2a^6M^2n/KT = 3.14 \times 10^{17}a^6M^2n/T \tag{7}$$

relative to the thermal energy KT. Hence, $\varepsilon_{ij} = 6.63 \times 10^{-3} \ll 1$ for $M = 10^6/4\pi$ A/m, $T = 300$°K, $a = 10^{-8}$ m, and $n \leq 10^{21}$ m^{-3}. Thus, the grains behave like an ideal gas at $T = 300°K$ for densities $n \leq 10^{21}\ m^{-3}$. The maximum binary interaction energy of the dipoles is ($r = 2a$)

$$\hat{\varepsilon}_{ij} = \mu_o \pi(\pi/6)^2 M^2 a^3/KT = 7.84 \times 10^{16} M^2 a^3/T \tag{8}$$

by Eq. (6). Accordingly, $\hat{\varepsilon}_{ij} \sim 10^0$ for $M = 10^6/4\pi$ A/m, $T = 300°K$, and $a = 10^{-8}$ m. Therefore, under typical conditions, some of the grains will always coagulate as a result of their dipole attraction.

These numerical illustrations indicate that the grains coagulate in their attracting inhomogeneous dipole fields $\vec{B}_s$ to larger macroparticles, in particular at low temperatures. In homogeneous external fields of moderate intensity, $B_o < 1$ Vsec/m^2, the coagulation leads to long chains consisting of up to 10^6 grains which are aligned parallel to $\vec{B}_o$. In thermal equilibrium, the chain length distribution is determined by the respective temperature T and grain density n.

KINETIC EQUATION

The diffusion of macroscopic ferromagnetic grains of radius a, mass m, moment of inertia I, angular momentum $\vec{L}$, and magnetic moment $\vec{p}$ in a nonmagnetic carrier medium F (gas or liquid), and an inhomogeneous magnetic field $\vec{B}$ represents, in general, a physically complex problem. The main complications are (i) the coupling of translational $(\vec{v})$ and angular $(\vec{\omega})$ velocities of each grain (Magnus effect) and (ii) the precession of the magnetic moment $\vec{p}$ about the local magnetic field $\vec{B}$ in case of incomplete alignment. The dynamical variables $\vec{L}$, $\vec{\omega}$, and $\vec{p}$ of a grain are interrelated by

$$\vec{L} = I\vec{\omega}, \quad \vec{p} = \gamma\vec{L} \tag{9}$$

where

$$d\vec{L}/dt = \vec{p} \times \vec{B}, \quad d\vec{p}/dt = \gamma\vec{p} \times \vec{B} \tag{10}$$

determine the precession of $\vec{L}$ and $\vec{p}$ in $\vec{B}$. The gyromagnetic factor is $\gamma = p_0 g/\hbar$ (g = Landé ratio, $p_0 = e\hbar/2m_e$ = Bohr magnetron), since the ferromagnetism of the grains is due to their electrons. The precession of $\vec{L}$ and $\vec{p}$ about the direction of $\vec{B}$ occurs with the modified Larmor frequency[7]

$$\omega_L = -\omega_B, \quad \omega_B = egB/2m_e > 0 \quad . \tag{11}$$

In view of Eq. (9), $\vec{L}$, $\vec{p}$, and $\vec{\omega}$ are equivalent dynamical variables for any grain. The distribution function of the translational $\vec{v}$ and angular velocities $\vec{\omega}$ of the grains is, therefore, at any point $(\vec{r},t)$ of the form

$$f = f(\vec{r},t,\vec{v},\vec{\omega}) \tag{12}$$

where

$$n = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f\, d^3\vec{v}\, d^3\vec{\omega}, \quad n\langle\vec{v}\rangle = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \vec{v} f\, d^3\vec{v}\, d^3\vec{\omega}, \ \ldots \tag{13}$$

are the macroscopic moments (grain density n, grain flux $n\langle\vec{v}\rangle$, ...) of $f(r,t,\vec{v},\vec{\omega})$. Changes of the distribution function are brought about by the force of the inhomogeneous $\vec{B}$ field on the magnetic moment $\vec{p}$, the translational friction force (relaxation time τ_1), the Magnus force, and the friction torque (relaxation time τ_2) on the grains:

$$\vec{F}_m = \vec{p}\cdot\nabla\vec{B} \quad , \tag{14}$$

$$\vec{F}_f = - m\vec{v}/\tau_1 \quad , \tag{15}$$

$$\vec{F}_c = - \kappa m_a \vec{v} \times \vec{\omega} \quad , \tag{16}$$

$$\vec{G}_f = - I\vec{\omega}/\tau_2 \quad . \tag{17}$$

The equivalent fluid mass of the grain volume is $m_a = 4\pi a^3 n_F m_F/3$. The Magnus force [Eq. (16)] couples the translational $\vec{v}$ and angular $\vec{\omega}$ motions of each grain such that a grain rolls aside if $\vec{v}$ is not parallel to $\vec{\omega}$.[8,9] The dimensionless correlation integral of $\vec{F}_c$ is of the order[9] $\kappa(K) \sim 10^0$. The frictional relaxation frequencies of $\vec{v}$ and $\vec{\omega}$ are in the free molecular region,[10] $K = \lambda_F/a \gg 1$:

$$\tau_1^{-1} = (8/3)a^2 n_F (2\pi m_F KT)^{1/2}(1 + \alpha_1\pi/8)/m \quad , \tag{18}$$

$$\tau_2^{-1} = (32/9)a^4 n_F (2\pi m_F KT)^{1/2}(1 + \alpha_2\pi/8)/I \quad , \tag{19}$$

where $0 < \alpha_{1,2} < 1$ are accommodation coefficients[10] (m_F = mass, n_F = density of carrier fluid). In gases with a m.f.p. $\lambda_F \sim 10^{-7}$ m, the Knudsen number is $K \sim 10^1$ for grains with radii $a \sim 10^{-8}$ m. The corresponding Stokes formulae[10] (η_F = viscosity of carrier fluid) hold in the continuum region, $K = \lambda_F/a \ll 1$:

$$\tau_1^{-1} = 6\pi\eta_F a/m, \qquad \tau_2^{-1} = 8\pi\eta_F a^3/I \quad . \tag{20}$$

For obvious physical reasons, the distribution function $f(\vec{r},t,\vec{v},\vec{\omega})$ of the grains satisfies a continuity equation in the $\{t,\vec{r},\vec{v},\vec{\omega}\}$ space. Consideration of the forces (14)-(16) and the torque (17) on the balance of grains which leave and enter the volume element $dV = d^3\vec{r}\, d^3\vec{v}\, d^3\vec{\omega}$ at the point $(\vec{r},\vec{v},\vec{\omega})$ in the time $dt \to 0$, yields the fundamental kinetic equation for the distribution function $f(\vec{r},t,\vec{v},\vec{\omega})$:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \vec{r}} \cdot (\vec{v}f) + \frac{\partial}{\partial \vec{v}} \cdot \left(\frac{\vec{p}\cdot\nabla\vec{B}}{m} f\right) + \frac{\partial}{\partial \vec{\omega}} \cdot (\dot{\vec{\omega}} f) =$$

$$\kappa \frac{m_a}{m}(\vec{v}\times\vec{\omega}) \cdot \frac{\partial f}{\partial \vec{\omega}} + \tau_1^{-1} \frac{\partial}{\partial \vec{v}} \cdot \left[\left(\vec{v} + \frac{KT}{m}\frac{\partial}{\partial \vec{v}}\right) f\right] + \tau_2^{-1} \frac{\partial}{\partial \vec{\omega}} \cdot \left[\left(\vec{\omega} + \frac{KT}{I}\frac{\partial}{\partial \vec{\omega}}\right) f\right] \tag{21}$$

where

$$\vec{B} = \mu_0\vec{H} + \mu_0 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \vec{p}\, f\, d^3\vec{v}\, d^3\vec{\omega}, \qquad \nabla \cdot \vec{B} = 0 \quad , \tag{22}$$

$$\vec{H} = -\nabla\Phi, \qquad \nabla \times \vec{H} = \vec{0} \quad , \tag{23}$$

and

$$\nabla^2\Phi = \nabla \cdot \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \vec{p}\, f\, d^3\vec{v}\, d^3\vec{\omega} \quad . \tag{24}$$

The Fokker-Planck equation (21) and the Poisson equation (24) represent a system of integro-differential equations for the determination of the distribution function $f(\vec{r},t,\vec{v},\vec{\omega})$ and the potential $\Phi(\vec{r},t)$ of the self-consistent magnetic field $\vec{H}(\vec{r},t)$, which has its sources in the distributed dipoles $\vec{p}$. The angular acceleration $\dot{\vec{\omega}}$ is defined by Eq. (10). Equations (10), (22), and (23) are auxiliary equations, assuming the absence of electric currents ($\nabla \times \vec{H} = \vec{0}$).

In Eq. (21) the interaction terms with coefficients κ, τ^{-1}, and τ_2^{-1} are responsible for the relaxation of f. In an external magnetic field $\vec{B}_o$, the thermal equilibrium distribution is given by ($\vec{p} = \gamma I\vec{\omega}$)

$$f_o = C_o(n_o, T_o, B_o) \exp[-(\frac{1}{2} m\vec{v}^2 + \frac{1}{2} I\vec{\omega}^2 - \vec{p}\cdot\vec{B}_o)/KT] \quad , \qquad (25)$$

if the nonideal field effects due to the dipole interactions are neglected (C_o = normalization constant). By Eq. (25), the average energies of translation and rotation are in thermal equilibrium T_o

$$\langle m\vec{v}^2/2\rangle = 3KT_o/2, \quad \langle I\vec{\omega}^2/2\rangle = 3KT_o/2 \quad . \qquad (26)$$

For vanishing interactions of the grains with the carrier medium, $\tau_{1,2} \to \infty$, the relaxation terms in Eq. (21) are absent. In this case, Eq. (21) has elementary, stationary ($\partial/\partial t \equiv 0$) solutions of the form

$$f_s = H(\varepsilon), \quad \varepsilon \equiv \frac{1}{2} m\vec{v}^2 + \frac{1}{2} I\vec{\omega}^2 - \int_{\vec{r}}(\vec{p}\cdot\nabla\vec{B}) \cdot d\vec{r} \quad , \qquad (27)$$

since

$$\partial f_s/\partial \vec{r} = -\, H'(\varepsilon)\vec{p}\cdot\nabla\vec{B}, \quad \partial f_s/\partial \vec{v} = H'(\varepsilon)m\vec{v} \quad , \tag{28}$$

$$\partial\cdot(\dot{\vec{\omega}} f_s)/\partial\vec{\omega} = H'(\varepsilon)I(\vec{\omega} - \gamma\vec{B}) \cdot \dot{\vec{\omega}} = 0 \quad , \tag{29}$$

where $\dot{\vec{\omega}} = \gamma\vec{\omega} \times \vec{B}$ is perpendicular to $(\vec{\omega} - \gamma\vec{B})$ by Eq. (10). In Eq. (27), $H(\varepsilon)$ is an arbitrary functional of ε and $H'(\varepsilon) \equiv dH(\varepsilon)/d\varepsilon$. The Magnus term in Eq. (21) vanishes since $\partial f_s/\partial\vec{v} \parallel \vec{v}$.

In general, nonequilibrium solutions of Eq. (21) are obtainable by means of perturbation theory. The latter assumes $f = f_o + \tilde{f}$ for $0 < \tau_{1/2} < \infty$, with $|\tilde{f}| \ll f_o$. It should be noted that the kinetic equation (21) implies that the grain component behaves like an ideal gas with an equation of state $p = nKT$.

DIFFUSION EQUATION

In Eq. (21), the Magnus force term causes grains with $\vec{v} \perp \vec{\omega}$ to diffuse somewhat slower than those with $\vec{v} \parallel \vec{\omega}$. As a result, the translational $(\vec{v})$ and rotational $(\vec{\omega})$ velocities are coupled, and the diffusion of grains becomes anisotropic. As known from the theory of the Senftleben effect[11-14] for molecules (with mechanical spin and magnetic moment) in a homogeneous magnetic field, the anisotropy of the diffusion coefficent decreases with increasing particle size.

Nondimensionalization of Eq. (21) indicates that the strength of the coupling of the translational $(\vec{v})$ and rotational $(\vec{\omega})$ motions by the Magnus force $\propto \vec{v} \times \vec{\omega}$ is determined by the relaxation frequency

$$\tau_c^{-1} = \kappa(2/3)^{1/2}(m_a/m)(3KT/I)^{1/2} \quad , \tag{30}$$

where $\omega_T = (3KT/I)^{1/2}$ is the thermal frequency of rotation, and $I = 3ma^2/5$ for spherical grains. Accordingly, one expects the diffusion anisotropies to be insignificant for coupling frequencies τ_c^{-1} which are small in comparison with the translational relaxation frequency τ_1^{-1}.

The diffusion equation for the grains follows from their continuity and motion equations in the carrier medium. The moments of the kinetic Eq. (21) for the corresponding dynamical variables $(m\vec{v})^0$ and $(m\vec{v})^1$ are

$$\partial n/\partial t + \nabla\cdot(n\langle\vec{v}\rangle) = 0 \quad , \tag{31}$$

$$\partial(nm\langle\vec{v}\rangle)/\partial t + \nabla\cdot(nm\langle\vec{v}\rangle\langle\vec{v}\rangle) = -\nabla(nKT) + n\langle\vec{p}\rangle\cdot\nabla\vec{B} - \tau_1^{-1}\, nm(\langle\vec{v}\rangle - \langle\vec{v}_F\rangle) \tag{32}$$

for

$$\tau_c^{-1} \ll \tau_1^{-1}(1 + \tau_2^{-1}/\tau_1^{-1})^{1/2} \sim \tau_1^{-1}, \quad \tau_2^{-1} < \tau_1^{-1} \quad . \tag{33}$$

Equation (33) is the condition for the quasi-isotropic diffusion approximation.[14] The sources for the momentum changes of the grains are the pressure gradient, the magnetic dipole force density, and the intercomponent friction force density between the grains and the carrier medium (F) with mean mass velocities $\langle\vec{v}\rangle$ and $\langle\vec{v}_F\rangle$, respectively.

The magnetic dipole force density in Eq. (32) results from Eq. (21), in accordance with the partial integration

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \vec{v}\ \partial\cdot[(\vec{p}\cdot\nabla\vec{B})f]/\partial\vec{v}\ d^3\vec{v}\ d^3\vec{\omega} = -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f\ \vec{p}\cdot\nabla\vec{B}\ d^3\vec{v}\ d^3\vec{\omega} = -n\langle\vec{p}\rangle\cdot\nabla\vec{B} \quad (34)$$

for particles with magnetic moment $\vec{p} = \gamma I\vec{\omega}$, and $f \sim \exp(-mv^2/KT) \to 0$ for $|\vec{v}| \to \infty$.

For diffusive or slow shock-free motions of the grains, the nonlinear inertia term $\nabla\cdot(nm\langle\vec{v}\rangle\langle\vec{v}\rangle)$ in Eq. (32) is neligible. In this approximation, elimination of $n\langle\vec{v}\rangle$ from Eqs. (31) and (32) yields the "hyperbolic" diffusion equation for the grain density field $n(\vec{r},t)$ in isothermal carrier media (T):

$$\partial^2 n/\partial t^2 + \tau_1^{-1}\ \partial n/\partial t + \tau_1^{-1}\nabla\cdot(n\langle\vec{v}_F\rangle) = c_T^2\nabla\cdot[\nabla n - n(\langle\vec{p}\rangle\cdot\nabla\vec{B})/KT] \quad , (35)$$

where

$$c_T = (KT/m)^{1/2} \quad (36)$$

is the thermal speed of the grains of mass m. Equation (35) represents a wave equation which propagates density perturbations $n(\vec{r},t)$ with a characteristic speed c_T and relaxation time τ_1.

In applications usually only large observation times are of interest, $t \gg \tau_1$. In this case, Eq. (35) can be reduced to the "parabolic" diffusion equation for grains in isothermal carrier media (T):

$$\partial n/\partial t + \nabla\cdot(n\langle\vec{v}_F\rangle) = D\nabla\cdot[\nabla n - n(\langle\vec{p}\rangle\cdot\nabla\vec{B})/KT] \quad (37)$$

where

$$D = \tau_1 c_T{}^2 = (KT/m)\tau_1 \quad (38)$$

is the diffusion coefficient in the quasi-isotropic approximation (33). Equation (37) propagates density perturbations with infinite maximum speed, since $c_T{}^2 \to \infty$ for $\tau_1 \to 0$ by Eq. (38).[15]

In Eqs. (35) and (37), $\nabla\cdot(n\langle\vec{v}_F\rangle)$ considers the convection of grains by the flow $\langle\vec{v}_F\rangle$ of the carrier medium (F). The self-consistent magnetic field $\vec{B}$ in these hyperbolic and parabolic diffusion equations is described by

$$\vec{B} = \mu_0\vec{H} + \mu_0 n\langle\vec{p}\rangle, \quad \nabla\cdot\vec{B} = 0, \quad (39)$$

$$\vec{H} = -\nabla\Phi, \quad \nabla\times\vec{H} = \vec{0}, \quad (40)$$

$$\nabla^2\Phi = \nabla\cdot(n\langle\vec{p}\rangle), \quad (41)$$

where

$$d\langle\vec{p}\rangle/dt = \gamma\langle\vec{p}\rangle \times \vec{B} \quad . \quad (42)$$

Equations (39)-(42) are the macroscopic (average) versions of Eqs. (22)-(24) and Eq. (10), respectively.

The presented diffusion theory for magnetic grains suspended in gaseous or liquid media (F) is applicable in the isotropic diffusion approximation (33), which requires that

$$\tau_c^{-1}/\tau_1^{-1} \sim (m_F/m)^{1/2} \ll 1, \quad K \gg 1, \quad (43)$$

$$\tau_c^{-1}/\tau_1^{-1} \sim (n_F m_F/\eta_F)(KT/m)^{1/2}a \ll 1, \; K \ll 1, \quad (44)$$

in the free molecular and continuum flow regions, respectively. For $K \gg 1$, the isotropy condition is satisfied since $m_F \ll m$ for macroscopic grains ($a \gg 10^{-10}$ m) of mass $m = (4\pi a^3/3)\rho$ (solid-state density $\rho > 10^3$ kg/m^3). For $K \ll 1$, the istropy condition is satisfied for dense gases, and also for typical liquids ($n_F m_F \sim 10^3$ kg/m^3, $\eta_F \sim 10^{-3}$ kg/msec, $T \sim 300$°K) for which $\tau_c{}^{-1}/\tau_1{}^{-1} \sim 10^{-6}/a^{1/2} \ll 1$ for $a > 10^{-10}$ m.

BOUNDARY-VALUE PROBLEM

In the classical theory of Smoluchowski,[16] noncharged and nonmagnetic colloidal particles of radii a and b $\gtrless$ a are shown to coagulate upon approaching their critical interaction sphere of radius $d = a + b$ by Brownian motion. When the grains are surrounded by a layer of solvent, the contact distance d is not exactly the sum of the grain radii a and b. If the macroscopic particles have a magnetic moment, the coagulation process is considerably enhanced by the drift motion of one dipole $\vec{p}_\alpha$ in the attractive, inhomogeneous magnetic field $\vec{B}_\beta$ of the other dipole $\vec{p}_\beta$. Experiments indicate that an external, homogeneous magnetic field B_0 directs the agglomeration of particles into conglomerates of the form of long chains parallel to $\vec{B}_0$.[5,6]

In order to provide an understanding of the coagulation of ferromagnetic grains in dilute suspension, the boundary-value problem for the spatial distribution $n(\vec{r})$ of similar "field" dipoles $\vec{p}_\alpha$ of radius $r = a$ in the magnetic field $\vec{B}_\beta(r)$ of a fixed "sink" dipole $\vec{p}_\beta$ of radius $r = b$ shall be analyzed, when a homogeneous magnetic field $\vec{B}_0$ is present. The latter is assumed to be parallel to the z-axis (Fig. 1) and is in spherical coordinates (r,θ,ϕ) given by

$$\vec{B}_0 = B_0 \{\cos\theta, -\sin\theta, 0\} \quad . \tag{45}$$

The sink dipole $\vec{p}_\beta$ is taken to be at the origin $\vec{r} = \vec{0}$ and is to be aligned with $\vec{B}_0$ (Fig. 1), so that in spherical coordinates

$$\vec{p}_\beta = p_\beta \{\cos\theta, -\sin\theta, 0\} \quad . \tag{46}$$

The magnetic field $\vec{B}_\beta = -\nabla(\mu_o \vec{p}_\beta \cdot \vec{r}/4\pi r^3)$ of the sink dipole $\vec{p}_\beta$ is by Eq. (46)

$$\vec{B}_\beta = A_\beta\ r^{-3}\{2\cos\theta,\ \sin\theta,\ 0\},\quad A_\beta \equiv \mu_o p_\beta/4\pi \quad . \tag{47}$$

The intensity of the total magnetic field $\vec{B}(\vec{r}) = \vec{B}_o + \vec{B}_\beta(\vec{r})$ at the position $\vec{r}$ from the origin (Fig. 1) is then

$$B(r,\theta) = [B_o^2 + 2B_o A_\beta r^{-3}(3\cos^2\theta - 1) + (A_\beta r^{-3})^2(3\cos^2\theta + 1)]^{1/2},\quad d \le r \le \infty \quad . \tag{48}$$

For saturated, ferromagnetic grains with quasi-instantaneous alignment[12] in the local magnetic field $\vec{B}(\vec{r})$, the magnetic dipole force is derivable from the potential[17] $\vec{p}_\alpha \cdot \vec{B}$,

$$\vec{p}_\alpha \cdot \nabla\vec{B} = \nabla(\vec{p}_\alpha \cdot \vec{B}),\quad \vec{p}_\alpha = p_\alpha \vec{B}/B \quad . \tag{49}$$

For isothermal systems, it is convenient to introduce the dimensionless potential

$$\psi(r,\theta) = [p_\alpha B(r,\theta) - p_\alpha B_o]/KT \tag{50}$$

with

$$\psi(r,\theta) \to 0,\ r \to \infty,\ 0 \le \theta \le \pi \quad , \tag{51}$$

$$\vec{p}_\alpha \cdot \nabla\vec{B}/KT = \nabla\psi \quad . \tag{52}$$

The carrier medium in which the magnetic grains are suspended is assumed to be at rest, $\langle\vec{v}_F\rangle = \vec{0}$. According to Eqs. (37), (50), and (52), the distribution $n(r,\theta)$ of field dipoles $\vec{p}_\alpha$ in the surrounding $d \le r \le \infty$ of the sink dipole $\vec{p}_\beta$ (Fig. 1), as modified by thermal diffusion $\propto\nabla n$, magnetic field drift $\propto\nabla\psi$, and coagulation at the contact sphere $r = d = a + b$ is, in the stationary state, determined by the elliptic boundary-value problem:

$$\nabla \cdot (\nabla n - n\nabla\psi) = 0, \quad d \leq r < \infty, \tag{53}$$

$$n(r = d, \theta) = 0, \quad 0 \leq \theta \leq \pi, \tag{54}$$

$$n(r = \infty, \theta) = n_0, \quad 0 \leq \theta \leq \pi, \tag{55}$$

where $n = n(r,\theta)$ due to azimuthal (ϕ) symmetry, and n_0 is the grain density at large distances $r \gg d$ from the sink dipole $\vec{p}_\beta$. For $\psi(r,\theta) \equiv 0$, Eqs. (53)-(55) reduce to Smoluchowski's boundary-value problem,[16] which has the simple solution $n(r,\theta) = n_0(1 - d/r)$.

The magnetic field drift term $\propto\nabla\psi$ and the complex r,θ-dependence (48) of the potential function $\psi(r,\theta)$ render the boundary-value problem (53)-(55) nontrivial. The ansatz

$$n(r,\theta) = N(r,\theta)e^{\psi(r,\theta)} \tag{56}$$

reduces Eq. (53) to the Laplace equation with a variable coefficient $\exp(\psi)$,

$$\nabla \cdot (e^{\psi}\nabla N) = 0 \quad . \tag{57}$$

The first integral of Eq. (57) is proportional to the grain flux $\vec{j} = -D \exp(\psi)\nabla N = -D(\nabla n - n\nabla\psi)$. Hence,

$$e^{\psi}\nabla N = \{(\partial S/\partial\theta)/r^2\sin\theta, \ -(\partial S/\partial r)/r\sin\theta, \ 0\} \quad . \tag{58}$$

can be derived from a stream function $S = S(r,\theta)$ so that $\nabla\cdot\vec{j} = 0$ and Eq. (57) is satisfied. Equation (58) yields

$$\nabla^2 N = -\frac{e^{-\psi}}{r^2 \sin\theta}\left(\frac{\partial\psi}{\partial r}\frac{\partial S}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial S}{\partial r}\right) \quad . \tag{59}$$

The right side of Eq. (59) suggests that the stream function is of the form

$$S(r,\theta) = \Omega(r,\theta)e^{\psi(r,\theta)} \tag{60}$$

which eliminates the variable coefficient $\exp(-\psi)$ from Eq. (59),

$$\nabla^2 N = -\frac{1}{r^2 \sin\theta}\left(\frac{\partial\psi}{\partial r}\frac{\partial\Omega}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial\Omega}{\partial r}\right) \quad . \tag{61}$$

In order to homogenize both boundary conditions (54) and (55), $N(r,\theta)$ is decomposed as

$$N(r,\theta) = n_o(1 - d/r) + g(r,\theta) \quad . \tag{62}$$

By Eqs. (53)-(55), (56), (61), and (62), the function $g(r,\theta)$ is determined by the boundary-value problem for a Poisson equation with homogeneous boundary conditions:

$$\nabla^2 g = Q(r,\theta), \quad d < r < \infty, \tag{63}$$

$$g(r = d,\theta) = 0, \qquad 0 \leq \theta \leq \pi, \tag{64}$$

$$g(r = \infty,\theta) = 0, \qquad 0 \leq \theta \leq \pi, \tag{65}$$

where

$$Q(r,\theta) = -\frac{1}{r^2 \sin\theta}\left(\frac{\partial\psi}{\partial r}\frac{\partial\Omega}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial\Omega}{\partial r}\right) \quad . \tag{66}$$

In the field-free case, the stream function is $S(r,\theta) = -n_o d\cos\theta$ for $\psi(r,\theta) = 0$. Hence, for $\psi(r,\theta) \neq 0$,

$$S(r,\theta) = -n_o d\, e^{\psi(r,\theta)}\cos\theta, \quad \Omega(r,\theta) = -n_o d\cos\theta, \tag{67}$$

and

$$Q(r,\theta) = -n_o dr^{-2}\,\partial\psi(r,\theta)/\partial r \quad , \tag{68}$$

with $r^2Q(r,\theta) = -n_o d\partial\Psi(r,\theta)/\partial r \to 0$ for $r \to \infty$ by Eqs. (48) and (50).

For the region outside of the interaction sphere, $d < r < \infty$, the Green's function of Eq. (63) and the homogeneous boundary conditions (64) and (65) is:

$$G(r,\theta;r',\theta') = -[R^{-1} - (d/r')\,R_o^{-1}]/4\pi \quad , \tag{69}$$

where

$$R(r,\theta;r',\theta') = [r^2 - 2rr'\cos(\theta-\theta') + r'^2]^{1/2} \tag{70}$$

$$R_o(r,\theta;r',\theta') = [(d^2/r')^2 - 2(d^2/r')r\cos(\theta-\theta') + r^2]^{1/2} \quad ,$$

with

$$R^{-1} = (d/r')R_o^{-1} \text{ for } r = d, \quad R = 0 \text{ for } r' = r,\ \theta' = \theta \quad . \tag{71}$$

The coordinates of the field and source points are designated by (r,θ) and (r',θ'), respectively. The Green's function (69) has the properties

$$G = \infty \text{ for } r'=r,\ \theta'=\theta; \quad G = 0 \text{ for } r=d,\ 0 \le \theta \le \pi; \quad G = 0 \text{ for } r=\infty,\ 0 \le \theta \le \pi \ . \tag{72}$$

The solution of Eqs. (63)-(65) is, in terms of the known functions $G(r,\theta;r',\theta')$ and $Q(r,\theta)$,

$$g(r,\theta) = \int_d^\infty \int_0^\pi Q(r',\theta')G(r,\theta;r',\theta')2\pi r'^2 \sin\theta' dr' d\theta' \quad . \tag{73}$$

Thus, we obtain from Eqs. (56), (62), and (69) the following analytical solution for the spatial distribution $n(r,\theta)$ of the field dipoles $\vec{p}_\alpha$ in the superimposed magnetic field $\vec{B}_\beta(r,\theta)$ of the sink dipole $\vec{p}_\beta$ and the external, homogeneous field $\vec{B}_o$:

$$n(r,\theta) = [n_o(1-d/r) + \int_d^\infty \int_0^\pi Q(r',\theta')G(r,\theta;r',\theta')2\pi r'^2 \sin\theta' dr' d\theta']e^{\psi(r,\theta)} \ . \tag{74}$$

This result shows that the distribution $n(r,\theta)$ of ferromagnetic grains is composed of the field-free solution multiplied by the Boltzmann factor, $n_o(1-d/r)\exp(\psi)$, and a source solution, $g(r,\theta)\exp(\psi)$, which vanishes at both boundaries $r = d$ and $r = \infty$.

The current density $j = -D(\nabla n - n\nabla\psi)$ of the magnetic grains α streaming to the interaction sphere $r = d$ around the sink dipole β is by Eq. (74),

$$\vec{j}(r,\theta) = -D[n_o dr^{-2}\vec{e}_r + \nabla \int_d^\infty \int_0^\pi Q(r',\theta')G(r,\theta;r',\theta')2\pi r'^2 \sin\theta' dr' d\theta']e^{\psi(r,\theta)} . \quad (75)$$

As an illustration, Figs. 2, 3, and 4 exhibit the dimensionless grain density $N(\rho,\theta) = n(r,\theta)/n_o$ versus the dimensionless distance $\rho = r/d \geq 1$ (from the source dipole β) for the angles $\theta = 0$; $\pi/4$; $\pi/2$, with $\varepsilon_o \equiv p_\alpha B_o/KT = 1$ and $\varepsilon \equiv p_\alpha \hat{B}_\beta/KT = 0.1$; 1; 10 as interaction parameters ($\hat{B}_\beta = \mu_o p_\beta/4\pi d^3$). It is seen that the field dipoles α are distributed anisotropically around the source dipole β, caused by its magnetic self-field $\vec{B}_\beta(r,\theta)$ and the external (homogenous) magnetic field $\vec{B}_o$, where $N(\rho,0) > N(\rho,\pi/4) > N(\rho,\pi/2)$. This anisotrophy effect increases strongly with increasing $\varepsilon \propto \hat{B}_\beta$ (Figs. 2, 3, 4), but increases only slightly with increasing $\varepsilon_o \propto B_o$ in the interval $0.1 \leq \varepsilon_o \leq 10$ (for the latter reason, only $\varepsilon_o = 1$ shown).

Comparison of Fig. 4 with Figs. 2 and 3 indicates that the magnetic grain distribution $N(\rho,\theta)$ changes qualitatively for large interaction parameters $\varepsilon \gtrsim 10$. The distributions $N(\rho,\theta)$ in Figs. 2 and 3 are of the "diffusion type," whereas the distribution $N(\rho,\theta)$ in Fig. 4 is controlled by the (α-β) dipole-dipole interaction. Since for $\varepsilon \gtrsim 10$ the energy $p_\alpha B_\beta$ of the field dipoles α in the magnetic field $\vec{B}_\beta$ of the source dipole β is much larger than the thermal energy KT, a large amassment of field dipoles α results in front of the reference dipole β (Fig. 4). In view of the binary interaction approximation, Fig. 4 is strictly applicable only to absolute α-dipole densities $n(r,\theta)$ for which the average binary (α-α') interaction parameter $\varepsilon_{\alpha\alpha'}$ is small [Eq. (7)].

In view of these observations, it is to be expected that the coagulation rate $\gamma_{\alpha\beta}$ of grains with magnetic moments $\vec{p}_{\alpha}$ and $\vec{p}_{\beta}$ depends (i) strongly on the magnetic dipole-dipole interaction paramters ε and (ii) on the interaction parameter ε_o of the dipoles with the external magnetic field $\vec{B}_o$. In the presence of a strong $\vec{B}_o$-field the coagulation rate $\gamma_{\alpha\beta}$ would obviously decrease with increasing $\varepsilon = \mu_o p_{\alpha} p_{\beta}/4\pi d^3 KT$ so that grains with large magnetic moments p_{α} and p_{β} coagulate at a slower rate than grains with smaller magnetic moments (in agreement with experiments).

COAGULATION IN MAGNETIC FIELD

The local density $n_{\alpha o}$ of particles (grains, chains) with magnetic moment p_α decreases due to coagulation with particles of magnetic moment p_β and local density $n_{\beta o}$ in accordance with the rate equation[16]

$$\partial n_{\alpha o}/\partial t = -\sum_{\beta} \gamma_{\alpha\beta} n_{\alpha o} n_{\beta o}. \tag{76}$$

The summation extends over all particle components $\beta \gtrless \alpha$ including $\beta = \alpha$. Equation (76) defines the binary coagulation coefficients $\gamma_{\alpha\beta}$[m^3/sec].

For the analytical calculation of the coagulation coefficients $\gamma_{\alpha\beta}$ use is made of the numerical result that the spatial distribution $n_\alpha(r,\theta)$ of α-dipoles around a β-dipole is given, in good approximation, by Boltzmann statistics as ($d_{\alpha\beta}$ = contact sphere radius for α-β coagulation)

$$n_\alpha(r,\theta) \cong n_{\alpha o}(1 - d_{\alpha\beta}/r)\ \exp[\psi_{\alpha\beta}(r,\theta)] \tag{77}$$

where

$$\psi_{\alpha\beta}(r,\theta) = -\frac{p_\alpha B_o}{KT} + \frac{p_\alpha B_o}{KT}\left(1 - \frac{r_\beta^3}{r^3}\right)\left[1 + 3\left(2\frac{r_\beta^3}{r^3} + 1\right)\left(\frac{r_\beta^3}{r^3} - 1\right)^{-2} \cos^2\theta\right]^{1/2} \tag{78}$$

and

$$r_\beta \equiv (A_\beta/B_o)^{1/3} = (\mu_o p_\beta/4\pi B_o)^{1/3} . \tag{79}$$

Equation (77) follows from the complete solution (74) by neglection of the $g(r,\theta)$ contribution (78) with vanishing boundary conditions [Eqs. (63)-(66)]. In this approximation, the flux $\Phi_{\alpha\beta} = 2\pi\int_0^\pi j_{r,\alpha\beta} r^2 \sin\theta d\theta$ of α-dipoles to the β-dipole is

$$\Phi_{\alpha\beta}(r) = - D_\alpha 2\pi n_{\alpha o} d_{\alpha\beta} \int_0^\pi \exp[\psi_{\alpha\beta}(r,\theta)] \sin\theta \, d\theta \tag{80}$$

since the α-dipole current density is $\vec{j}_{\alpha\beta} = - D_\alpha(\nabla n_\alpha - n_\alpha \nabla\psi_{\alpha\beta}) \cong - D_\alpha n_{\alpha o} d_{\alpha\beta} r^{-2} \exp(\psi_{\alpha\beta})\vec{e}_r$ by Eq. (77). The number of α-dipoles which reach, per unit time, the contact sphere $r = d_{\alpha\beta}$ of one β-dipole is

$$n_{\alpha o}\,\gamma_{\alpha\beta} = - C_{\alpha\beta}\Phi_{\alpha\beta}(r = d_{\alpha\beta}), \qquad C_{\alpha\beta} \cong 1,\ \alpha \ll \beta; \qquad C_{\alpha\beta} \cong 2,\ \alpha = \beta. \tag{81}$$

$C_{\alpha\beta}$ is an accommodation coefficient, which considers that not only the α-dipole but also the β-dipole is in thermal motion.[16] Equations (80) and (81) yield for the coagulation coefficient in an external (homogeneous) magnetic field the formula:

$$\gamma_{\alpha\beta} = 4\pi C_{\alpha\beta} d_{\alpha\beta} D_\alpha \exp(-\frac{p_\alpha B_o}{KT}) \int_0^1 \exp\{\frac{p_\alpha B_o}{KT}(1 - \rho_{\alpha\beta}^{-3})[1+3\,\frac{(2\rho_{\alpha\beta}^3+1)}{(\rho_{\alpha\beta}^3-1)^2}\,\sigma^2]^{1/2}\}d\sigma \tag{82}$$

where

$$\rho_{\alpha\beta} \equiv d_{\alpha\beta}/r_\beta = (4\pi d_{\alpha\beta}^3 B_o/\mu_o p_\beta)^{1/3} \tag{83}$$

and $\sigma = \cos\theta$. For the practical evaluation of the integral in Eq. (82) it is noted that the dimensionless parameter $\rho_{\alpha\beta}{}^3$ is very large even for moderate external $\vec{B}_o$-fields, e.g., $\rho_{\alpha\beta}{}^3 \sim 10^4$ for $d_{\alpha\beta} \sim 10^{-7}$m, $p_\beta \sim 10^{18}$Am^2, and $B_o = 1$ Vsec/m^2. Accordingly,

$$\gamma_{\alpha\beta} \cong 4\pi C_{\alpha\beta} d_{\alpha\beta} D_\alpha \exp(-\frac{p_\alpha B_o}{KT\rho_{\alpha\beta}^3})[1 + \sum_{n=1}^{\infty}\frac{(-6)^n}{1\cdot 3\cdots(2n+1)}\,(\frac{p_\alpha B_o}{KT\rho_{\alpha\beta}^3})^n\},\quad \rho_{\alpha\beta}^3 \gg 1, \tag{84}$$

or

$$\gamma_{\alpha\beta} \cong 4\pi C_{\alpha\beta} d_{\alpha\beta} D_\alpha \exp(-\frac{\mu_o p_\alpha p_\beta}{4\pi d_{\alpha\beta}^3 KT})[1 + \sum_{n=1}^{\infty}\frac{(-6)^n}{1\cdot 3\cdots(2n+1)}\,(\frac{\mu_o p_\alpha p_\beta}{4\pi d_{\alpha\beta}^3 KT})^n],$$

$$B_o \gg \mu_o p_\beta/4\pi d_{\alpha\beta}^3 \tag{85}$$

Although Eq. (82) predicts a B_o-dependence in general, the coagulation coefficient $\gamma_{\alpha\beta}$ is no longer B_o-dependent in the limit of large B_o-fields, $\rho_{\alpha\beta}{}^3 \gg 1$ or $B_o \gg \mu_o p_\beta / 4\pi d_{\alpha\beta}{}^3$. Equation (85) shows that $\gamma_{\alpha\beta}$ decreases exponentially with increasing ratio of dipole interaction energy $\mu_o p_\alpha p_\beta / 4\pi d_{\alpha\beta}{}^3$ to thermal energy KT. Hence, grains or chains with large dipole moments coagulate slower than those with small dipole moments, in the presence of a strong external magnetic field $\vec{B}_o$ [Eq. (85)].

In the absence of an external magnetic field, $\vec{B}_o = 0$, we have $\rho_{\alpha\beta} = 0$ by Eq. (83). In this case, Eq. (82) gives for the coagulation coefficient

$$\gamma_{\alpha\beta} = 4\pi C_{\alpha\beta} d_{\alpha\beta} D_\alpha \int_0^1 \exp\left[\frac{\mu_o p_\alpha p_\beta}{4\pi d_{\alpha\beta}^3 KT}(1 + 3\sigma^2)^{1/2}\right] d\sigma \ , \ B_o = 0 \quad . \qquad (86)$$

Since $0 \leq 3\sigma^2 \leq 3$ for $0 \leq \sigma \leq 1$, the integral in Eq. (86) can no longer be approximated by a rapidly converging series. For particles without dipole interactions (rigid spheres), $\rho_{\alpha,\beta} = 0$, Eq. (86) reduces to Smoluchowski's formula $\gamma_{\alpha\beta} = 4\pi C_{\alpha\beta} d_{\alpha\beta} D_\alpha$.[16]

The integral functional in Eq. (86) indicates that the coagulation rate $\gamma_{\alpha\beta}$ increases essentially exponentially with increasing ratio of dipole interaction energy $\mu_o p_\alpha p_\beta / 4\pi d_{\alpha\beta}{}^3$ to thermal energy KT, if an external magnetic field is not present. Thus, in the case $B_o = 0$, $\gamma_{\alpha\beta}$ is larger than in the presence of a strong external magnetic field, $B_o \gg \mu_o p_\beta / 4\pi d_{\alpha\beta}{}^3$. In the latter case, $\gamma_{\alpha\beta}$ decreases exponentially with the same energy ratio [Eq. (85)].

The stabilizing effect of an external magnetic field B_o concerning the decay of a magnetic aerosol by coagulation is experimentally established. In particular, homogeneous magnetic fields $B_o \gtrsim 10^{-2}$ Vsec/m are applied to the chambers of ovens for the generation of ferromagnetic aerosols, in order to reduce the coagulation rate and to align the ferromagnetic colloid chains. By Eq. (76) the relaxation time $\tau_\alpha{}^c$ for coagulative decay of the density $n_{\alpha o}$ of dipole particles with magnetic moment p_α is

$$\tau_\alpha^c = 1/\sum_\beta (\tau_{\alpha\beta}^c)^{-1} \;, \quad \tau_{\alpha\beta}^c = 1/\gamma_{\alpha\beta} n_{\beta o} \quad . \tag{87}$$

Another method of reducing the decay rate of ferromagnetic aerosols is to spray the ferromagnetic particles with electric charges. The resulting Couloumb repulsion of the magnetic particles effectively reduces coagulation as has been demonstrated experimentally.[17]

The presented theory should be considered as a first step towards a quantitative understanding of the coagulation of magnetic aerosols in external magnetic fields. At higher aerosol densities, many - dipole interactions have to be considered (in addition to the binary dipole interactions) in the analysis of the coagulation coefficients. In principle, this can be accomplished by means of the kinetic equations (21)-(24) or the corresponding macroscopic transport equations (37)-(42), with self-consistent magnetic dipole interactions.

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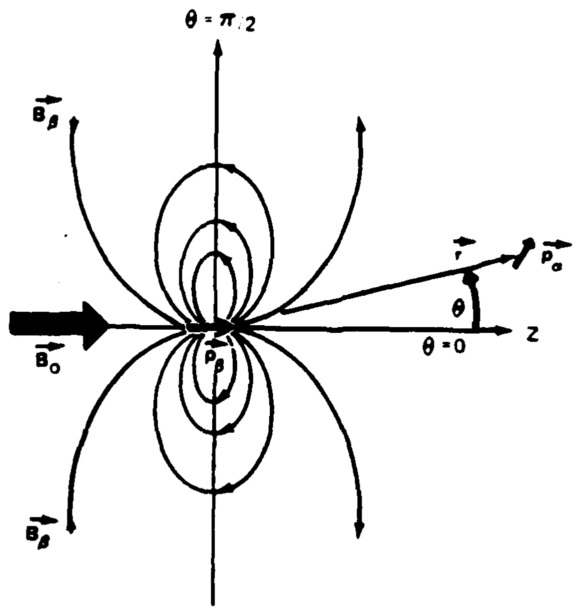

FIG. 1: Field dipole $\overrightarrow{p_\alpha}$ in the magnetic field $\vec{B}(r,\theta)$ of a source dipole $\vec{p}_\beta$ and a homogeneous magnetic field $\vec{B}_0$.

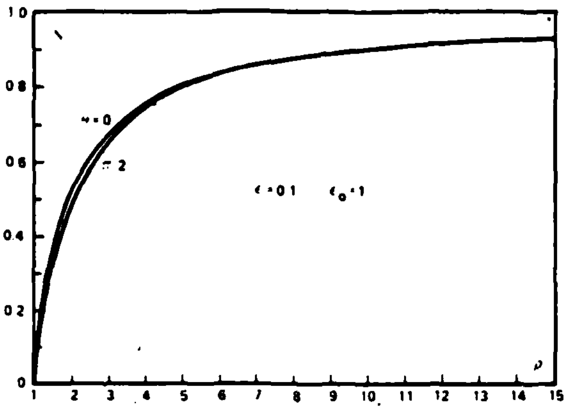

FIG. 2: Dimensionless grain density $N(\rho,\theta)$ versus $\rho = r/d \geq 1$ for $\theta = 0$; $\pi/4$; $\pi/2$, $\varepsilon_0 = 1$, and $\varepsilon = 0.1$.

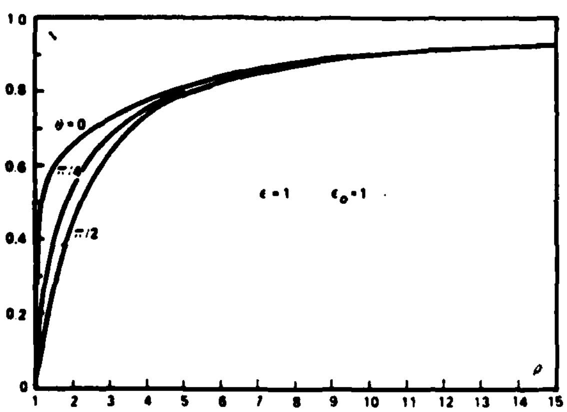

FIG. 3: Dimensionless grain density $N(\rho,\theta)$ versus $\rho = r/d \geq 1$ for $\theta = 0$; $\pi/4$; $\pi/2$, $\varepsilon_0 = 1$, and $\varepsilon = 1$.

FIGURE CAPTIONS

FIG. 1: Field dipole $\vec{p}_\alpha$ in the magnetic field $\vec{B}(r,\theta)$ of a source dipole $\vec{p}_\beta$ and a homogeneous magnetic field $\vec{B}_o$.

FIG. 2: Dimensionless grain density $N(\rho,\theta)$ versus $\rho = r/d \geq 1$ for $\theta = 0$; $\pi/4$; $\pi/2$, $\varepsilon_o = 1$, and $\varepsilon = 0.1$.

FIG. 3: Dimensionless grain density $N(\rho,\theta)$ versus $\rho = r/d \geq 1$ for $\theta = 0$; $\pi/4$; $\pi/2$, $\varepsilon_o = 1$, and $\varepsilon = 1$.

FIG. 4: Dimensionless grain density $N(\rho,\theta)$ versus $\rho = r/d \geq 1$ for $\theta = 0$; $\pi/4$; $\pi/2$, $\varepsilon_o = 1$, and $\varepsilon = 10$.

APPENDIX C

STATISTICAL DISTRIBUTION OF RANDOM ELECTRIC FIELDS IN CHARGED PARTICLE GASES

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ABSTRACT

The temperature (T) and density (n) dependent probability distribution $W = W(\vec{E};T,n)$ of the collective electric fields $\vec{E}$ in an ideal gas of charged particles each carrying the same charge e (electrons: $e = -e_o < 0$; ions: $e = Ze_o \gtrsim 0$) is calculated from first principles of statistical mechanics. It is shown that (i) the collective electric fields act at distances larger than the characteristic repulsion distance $D = (KT/4\pi ne^2)^{1/2}$ of like charged particles, and (ii) the average collective electric field is $E_W = (12\pi nKT)^{1/2}$ for ideal gases of particles of the same charge e. Thus, in a thermal equilibrium gas of like charged particles, the longitudinal microfields are considerably stronger than in a quasi-neutral plasma, since in the latter the random field effects of the negative electrons and positive ions nearly compensate each other. Finally, the interrelation between (average) kinetic, interaction, collective field, and electric self energies is discussed for charged particle gases.

INTRODUCTION

The probability distribution of the stochastic electric fields $\vec{E}$ produced by charges $e \lessgtr 0$ of one kind in random thermal motion is of considerable interest since one-component charged particle gases such as electron or ion gases are employed in many technical applications of physical electronics (von Ollendorff 1957). In a macroscopically homogeneous charged particle gas of density n and temperature T, collective particle interactions occur at distances $r > D$, since the minimum wave length of the random, thermally excited longitudinal charge waves is of the order $\lambda_m \sim D$ of the characteristic repulsion distance for particles with the same charge e,

$$D = (KT/4\pi ne^2)^{1/2} \quad . \tag{1}$$

By means of Poisson's equation, the random collective field amplitudes E_i in an arbitrary direction "i" can be estimated from the random particle density n as

$$E_i/D \sim \pm 4\pi e n \quad . \tag{2}$$

Equation (1) and (2) show that an equipartition between random collective field and thermal energies exists on the average in a gas of like charges,

$$\langle E_i^2/8\pi \rangle = nKT/2 \quad . \tag{3}$$

A rigorous derivation of Eq. (3) based on the Markov method has been given by Mints (1957) for a gas of electrons in thermal equilibrium.

By the fundamental axiom of statistical mechanics of ideal systems in thermal equilibrium, all equilibrium distributions can be derived without consideration of the interactions which bring about the equilibrium (Tolman 1938). By extending this principle for many-particle systems with discrete energies to continuous media with random energy densities $u = \vec{E}(\vec{r},t)^2/8\pi$, we derive the probability distribution $W(\vec{E})$ of the collective electric fields $\vec{E}(\vec{r},t)$ in charged particle gases. These are assumed to be "ideal", i.e., the Coulomb repulsion energy $e^2/\bar{r}$ is assumed to be small compared with the thermal energy KT,

$$\gamma = e^2 n^{1/3}/KT = 1.670\times 10^{-3} Z^2 n^{1/3} T^{-1} << 1 \quad . \tag{4}$$

Among the results reported, it is shown that the longitudinal microfields in gases of like charged particles are much larger than those of quasi-neutral plasmas, in which the random electric fields of the negative electrons and positive ions nearly compensate each other.

PROBABILITY DISTRIBUTION

Subject of the considerations is a homogeneous gas of volume Ω containing n charges e per unit volume. In thermal equilibrium, the kinetic energy density of the charges of mass m and velocities $\vec{v}_\mu$ is given by

$$\langle \sum_{\mu=1}^{N} \frac{1}{2} m\vec{v}_\mu^{\,2} \rangle = \frac{3}{2} n\Omega KT \quad , \quad N = n\Omega \quad . \tag{5}$$

During the random thermal motions of the charged particles, a continuous transformation of kinetic particle energy into potential electric energy occurs, and vice versa, due to the particle interactiond through their longitudinal Coulomb fields (transverse electromagnetic interactions are negligible for $mc^2 \gg KT$). As has been shown first by Mints (1957), equipartition of average random electric and kinetic energies exists in statistical equilibrium [for a thermodynamic derivation, see Eq. (46)] of a gas of particles with the same charge,

$$\langle \vec{E}^2/8\pi \rangle = \frac{3}{2} nKT \quad . \tag{6}$$

The electric field $\vec{E}(\vec{r},t)$ produced collectively by the charges at any point $\vec{r} \in \Omega$ and the field energy density $u = \vec{E}(\vec{r},t)^2/8\pi$ fluctuate with time t about the average values $\langle \vec{E} \rangle = \vec{0}$ and $\langle \vec{E}^2/8\pi \rangle \neq 0$, respectively. The proposed problem is to derive the probability $W(\vec{E})d^3\vec{E}$ for finding the collective field fluctuation $\vec{E}$ in the volume element $d^3\vec{E} = dE_x dE_y dE_z$ about the point $\vec{E} = (E_x,E_y,E_z)$ of the field space subject to the thermal equilibrium conditions (5) and (6).

In order to determine experimentally the collective microfield distribution $W(\vec{E}) = W(\vec{E}^2/8\pi)$ in a homogeneous and isotropic gas of charged particles, one would have to measure the fluctuating field $\vec{E}$ or the fluctuating energy density $\vec{E}^2/8\pi$ in the vicinity $\Delta^3\vec{r}$ of a fixed field point $\vec{r}\epsilon\Omega$ at consecutive times $t_\nu = \nu\Theta_\nu$,

$\nu = 1,2,3,\ldots N$, within experimental errors $\Delta t_\nu << \Theta$, where Θ_ν is a time interval which is large compared with the correlation time of the stochastic field so that $\langle\vec{E}(\vec{r},t)\vec{E}(\vec{r},t+\Theta_\nu)\rangle = 0$ (within these limitations, the magnitude of Θ_ν may be changed from one measurement ν to the next $\nu + 1$). In a large number of such measurements, $N \to \infty$, the energy density $\vec{E}_1^2/8\pi$ would be observed N_1 times, ..., the energy density $\vec{E}_\alpha^2/8\pi$ would be observed N_α times, etc., where $\vec{E}_\alpha^2/8\pi$ means an experimental value measured with an error $\Delta(\vec{E}_\alpha^2/8\pi)$. The resulting step-shaped energy distribution $N_\alpha = N_\alpha(\vec{E}_\alpha^2/8\pi)$ is represented by the partition

$$\begin{array}{ccccccccc} N_1 & N_2 & N_3 & \ldots & N_\alpha & \ldots & N_M \\ \vec{E}_1^2/8\pi & \vec{E}_2^2/8\pi & \vec{E}_3^2/8\pi & \ldots & \vec{E}_\alpha^2/8\pi & \ldots & \vec{E}_M^2/8\pi \end{array} \tag{7}$$

where

$$N_1 + N_2 + N_3 + \ldots N_\alpha + \ldots N_M = N \quad , \tag{8}$$

$$N_1\vec{E}_1^2/8\pi + N_2\vec{E}_2^2/8\pi + N_3\vec{E}_3^2/8\pi + \ldots N_\alpha\vec{E}_\alpha^2/8\pi + \ldots N_M\vec{E}_M^2/8\pi = N\langle\vec{E}^2/8\pi\rangle. \tag{9}$$

N is the total number of measurements ($N \to \infty$) and $N\langle\vec{E}^2/8\pi\rangle$ is the total field energy density measured in the N independent observations. The entire energy density $N\langle\vec{E}^2/8\pi\rangle$ can be distributed in a large number Π of ways over sets $\{N_\alpha\}_N$ of numbers N. By elementary combinatorics (Tolman 1938),

$$\Pi = N! \; / \; N_1!\, N_2!\, N_3! \ldots N_\alpha! \ldots N_M! \quad . \tag{10}$$

The energy distribution $N_\alpha(\vec{E}_\alpha^2/8\pi)$ observed in statistical equilibrium is the most probable one. Thus, $N_\alpha(\vec{E}_\alpha^2/8\pi)$ is determined by the condition for a maximum of i) the number Π of realizations or ii) the entropy $S \sim \ln\Pi$, subject to the constraints (8) and (9).

Accordingly, we determine the probability distribution $N_\alpha(\vec{E}_\alpha^2/8\pi)$ from the maximum of the function $\ln\Pi \equiv f(N_\alpha)$,

$$\ln\Pi = \mathcal{N}(\ln\mathcal{N} - 1) - \sum_{\alpha=1}^{M} N_\alpha(\ln N_\alpha - 1) \tag{11}$$

with

$$\sum_{\alpha=1}^{M} N_\alpha = \mathcal{N} \quad , \qquad \mathcal{N} \to \infty \quad , \tag{12}$$

$$\sum_{\alpha=1}^{M} N_\alpha \vec{E}_\alpha^2/8\pi = \mathcal{N}\frac{3}{2}\,nKT \quad , \qquad \mathcal{N} \to \infty \quad , \tag{13}$$

as constraints. Eq. (12) holds by definition of $\mathcal{N}$, whereas Eq. (13) holds for a large number $\mathcal{N}$ of measurements and the average energy density $\langle\vec{E}^2/8\pi\rangle$ of Eq. (6). Addition of the constraints (12) and (13) multiplied by the Lagrangian multipliers $-\lambda$ and $-\mu$ to Eq. (11) leads to the compact maximum conditions for $\ln\Pi$,

$$\partial F(N_\alpha)/\partial N_\alpha = 0 \quad , \quad \partial^2 F(N_\alpha)/\partial N_\alpha^2 < 0 \quad , \quad \alpha = 1,2,\ldots M \quad , \tag{14}$$

where

$$F(N_\alpha) = \mathcal{N}(\ln\mathcal{N} - 1) - \sum_{\alpha=1}^{M} N_\alpha(\ln N_\alpha - 1) - \lambda\sum_{\alpha=1}^{M} N_\alpha - \mu\sum_{\alpha=1}^{M} N_\alpha \vec{E}_\alpha^2/8\pi \quad . \tag{15}$$

The solution of Eq. (14) gives the distribution N_α of the "discrete" energy densities $\vec{E}_\alpha^2/8\pi$ in the form

$$N_\alpha = Ae^{-\mu\vec{E}_\alpha^2/8\pi} \quad , \qquad A \equiv e^{-(1+\lambda)} \quad . \tag{16}$$

Henceforth, the subscript α is dropped since $\vec{E}_\alpha$ can be any point $\vec{E}$ in the field space. The dimensional constants $A(\lambda)$ and μ are then given by the normalization conditions (12) and (13),

$$A\int_0^\infty e^{-\mu\vec{E}^2/8\pi}\,4\pi E^2 dE = \mathcal{N} \quad , \tag{17}$$

$$A \int_0^{\infty} (\vec{E}^2/8\pi) e^{-\mu \vec{E}^2/8\pi} 4\pi E^2 dE = N \frac{3}{2}\, nKT \quad , \quad (18)$$

as

$$A = (8\pi^2 nKT)^{-3/2} N \quad , \quad \mu = 1/nKT \quad . \quad (19)$$

For this normalization, which still contains the number N of measurements, the probability distribution (16) for the microfield energy density is

$$W_N(\vec{E}^2/8\pi) = \frac{N}{(8\pi^2 nKT)^{3/2}}\, e^{-\vec{E}^2/8\pi nKT} \quad . \quad (20)$$

In theoretical applications, one is interested in the probability $dP = W(\vec{E})d^3\vec{E}$ for finding a microfield $\vec{E}$ in the volume element $d^3\vec{E}$ about the point $\vec{E}$ of the field space, with the normalization $\int dP = 1$. The corresponding distribution function $W(\vec{E})$ of the collective microfield $\vec{E}$ is obtained by renormalization ($N \to 1$):

$$W(\vec{E}) = (8\pi^2 nKT)^{-3/2} e^{-\vec{E}^2/8\pi nKT} \quad . \quad (21)$$

This fundamental distribution has the form of a Gaussian, i.e., all its moments exist, e.g.,

$$\langle \vec{E}^0 \rangle = \iiint_{-\infty}^{+\infty} \vec{E}^0 W(\vec{E})\, d^3\vec{E} = \vec{1} \quad , \quad (22)$$

$$\langle \vec{E}^1 \rangle = \iiint_{-\infty}^{+\infty} \vec{E}^1 W(\vec{E}) d^3\vec{E} = \vec{0} \quad , \quad (23)$$

$$\langle \vec{E}^2 \rangle = \iiint_{-\infty}^{+\infty} \vec{E}^2 W(\vec{E}) d^3\vec{E} = 12\pi nKT \quad . \quad (24)$$

The most probable (E_M) and the r.m.s. (E_W) collective microfields are by Eqs. (21) and (24)

$$E_M = (8\pi nKT)^{1/2} \quad , \quad (25)$$

$$E_W = (12\pi nKT)^{1/2} \quad . \quad (26)$$

For considerations concerning the fluctuation of the collective microfield $\vec{E}(t)$ at a point $\vec{r}\epsilon\Omega$ with time t, temporal averages can be defined by

$$\overline{|\vec{E}|} = \lim_{\tau\to\infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} |\vec{E}(t)|dt \quad , \quad (27)$$

$$\overline{\vec{E}^2} = \lim_{\tau\to\infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} \vec{E}(t)^2 dt \quad . \quad (28)$$

The fluctuation of $\vec{E}(t)$ is defined by $\Delta\vec{E}(t) = \vec{E}(t) - \overline{\vec{E}(t)}$ with $\overline{\vec{E}(t)} = \vec{0}$. In stationary equilibrium, the time averages are identical with the ensemble averages. By Eq. (21), the mean square (temporal) fluctuation of $\vec{E}(t)$ is

$$\overline{\Delta E^2} = \overline{\vec{E}^2} - \overline{|\vec{E}|}^{\,2} = (3 - \frac{8}{\pi})4\pi nKT \quad . \quad (29)$$

TABLE I compares the r.m.s. field E_W and the r.m.s. fluctuation $(\overline{\Delta E^2})^{1/2}$ of the collective microfield with the nearest neighbor Coulomb field $E_o = 2\pi(4/5)^{2/3} \times |e|n^{2/3}$ in dependence of the particle density n for an ideal electron gas ($\gamma << 1$, $T = 10^4$°K). It is seen that E_W and $(\Delta E^2)^{1/2}$ are one to two orders of magnitude larger than E_o in the range of ideal gas densities $n < 10^{18}cm^{-3}$. For these reasons, the Coulomb field E_o represents a small contribution to the microfield in ideal gases of charged particles. The result $E_W >> E_o$ is readily understandable since for ideal conditions

$$E_o^2/E_W^2 = (\pi/3)(4/15)^{4/3} \cdot \frac{e^2 n^{1/3}}{KT} < \gamma << 1 \quad . \quad (30)$$

The probability for observing a collective microfield with intensity $E = |\vec{E}|$ in the range between E and E + dE is $P(E)dE = W(\vec{E})4\pi E^2 dE$, where $W(\vec{E})$ is given by Eq. (21). The maximum of the probability density P(E) is $P(E_M) = 4\exp^{-1}(8\pi^2 nKT)^{-1/2}$ by Eq. (25). Accordingly, the normalized probability density is $P(E)/P(E_M) = \times(E^2/ 8\pi nKT)\exp[1 - E^2/8\pi nKT] \leq 1$. Figure 1 presents $P(E)/P(E_M) \equiv P(E^2/8\pi nKT)$ versus $E^2/8\pi nKT$. This distribution is a displaced Gaussian with a maximum

$P(E_M^2/8\pi nKT) = 1$ at $E = E_M$. The most probable electric field $E_M = (8\pi nKT)^{1/2}$ increases proportional $(nT)^{1/2}$ with increasing nT values. The increasing quantitative importance of the collective microfield E in ideal charged particle gases with higher pressures $p = nKT$ is obvious.

On the other hand, in a quasi-neutral electron (e) - ion (i) plasma ($n_e = Zn_i$), the microfield energy U_p differs by a factor of order $(n_e D_\pm^3)^{-1}$ from the microfield energy $E_W^2/8\pi$ [Eq. (26)] of the charged particle gas, where $D_\pm = [KT/4\pi(n_e e_e^2 + n_i e_i^2)]^{1/2} \neq D$ is the Debye shielding radius (Debye and Hueckel 1923). Since $(n_e D_\pm^3)^{-1} \ll 1$ for ideal conditions, the microfields in plasmas are small in comparison with those in ideal gases of like charged particles. In a plasma, the random electric fields of the negative electrons and positive ions compensate each ether nearly completely. This incomplete statistical compensation is the physical reason why $0 < U_p \ll E_W^2/8\pi$.

ENERGY RELATIONS

A gas of charged particles in thermal equilibrium at a temperature T exhibits various macroscopic energies, the average kinetic energy $\langle\mathcal{K}\rangle = \frac{3}{2}nKT\Omega$, the average electric field energy $\langle U\rangle = \langle\vec{E}^2/8\pi\rangle\Omega$, the average interaction energy $\langle\Phi\rangle$ and selfenergy $\langle\Psi\rangle$ of the charged particles. In order to derive the interrelation between these energies, the formation of the gas by an electric charging process is considered. For this purpose, we assume that the charged particles are initially dispersed at infinity where they have only selfenergies. The gas is then built up by moving one charge after the other from infinity into the volume Ω, which requires work against the resulting Coulomb field of the charges already present in Ω. The thermodynamics of the charging process is illustrated by i) a reversible isothermal and ii) an adiabatic or isotropic model.

The electric charging work expanded in moving N charges e against their collective Coulomb field from infinity into the (finite) volume Ω is (* designates exclusion of terms with $\mu = \nu$)

$$A = \frac{1}{2}\sum_{\mu=1}^{N}{}^{*}\sum_{\nu=1}^{N}{}^{*} e^2 |\vec{r}_\mu - \vec{r}_\nu|^{-1} \quad , \quad \vec{r}_{\mu,\nu} \in \Omega \quad . \tag{31}$$

where $\vec{r}_\mu$ ($\vec{r}_\nu$) are the position vectors of the μ-th (ν-th) charge e in the volume Ω, respectively. The collective microfield of the N charges at a field point $(\vec{r},t)$ is the superposition

$$\vec{E}(\vec{r},t) = \sum_{\mu=1}^{N} \vec{E}_\mu(\vec{r},t) \tag{32}$$

where $\vec{E}_\mu(\vec{r},t)$ are the individual Coulomb fields produced at the field point $(\vec{r},t)$ by the μ-th charge. By Eq. (32), the electric field energy of the gas Ω is

$$U = \frac{1}{8\pi}\iiint_\Omega \vec{E}(\vec{r},t)^2 d^3\vec{r} = \Phi + \Psi \tag{33}$$

where

$$\Phi = \frac{1}{8\pi}\sum_{\mu=1}^{N}{}^{*}\sum_{\nu=1}^{N}{}^{*}\iiint_\Omega \vec{E}_\mu \cdot \vec{E}_\nu d^3\vec{r} \quad , \tag{34}$$

$$\Psi = \frac{1}{8\pi}\sum_{\mu=1}^{N}\iiint_\Omega (\vec{E}_\mu)^2 d^3\vec{r} \quad , \tag{35}$$

are the (e-e) interaction energy and the (e)selfenergy of the charged particle gas, respectively. Comparison of Eq. (31) with Eq. (33) reveals the interrelation

$$U - \Psi = \Phi = A \quad . \tag{36}$$

Thus, we see that the field energy U is the sum of the interaction energy Φ and the selfenergy Ψ [Eq. (35)]. The charging work A leads to an increase of the interaction part Φ of the field energy U [Eq. (36)]. The selfenergy Ψ of the charges is independent of the spatial locations of the charges, i.e., Ψ is the same before and after the charging process.

Another independent energy relation is obtained by multiplication of the coupled Newtonian equations for the accelerations $d^2\vec{r}_{\mu,\nu}(t)/dt^2$ of the μ-th charge and the ν-th charge by their respective velocities $\vec{v}_{\mu,\nu}(t) = d\vec{r}_{\mu,\nu}(t)/dt$ and subsequent summation over all particles μ and ν. The resulting expression can be brought into the form $d(K + \Phi)/dt = 0$, which demonstrates that the sum of kinetic (K) and interaction (Φ) energies is an invariant H_o,

$$K + \Phi = H_o \tag{37}$$

where

$$K = \sum_{\mu=1}^{N}\frac{1}{2}m\vec{v}_\mu^2 \tag{38}$$

and $\Phi = A$ is defined in Eq. (31). Eq. (37) expresses the conservation of kinetic K and interaction Φ energies in a gas of charged particles which interact by longitudinal Coulomb fields.

The thermodynamic functions of the gas depend in general on the volume Ω, the number N of particles in Ω, and the particle averages of the random kinetic energies $\frac{1}{2}m\vec{v}^2$ and the random field energy densities $\vec{E}^2/8\pi$. Accordingly, we assume $U^{th} = U^{th}(T,\varepsilon,N)$ for the thermal energy and $S = S(T,\varepsilon,N)$ for the entropy, where

$$3KT/2 = \langle\frac{1}{2}m\vec{v}^2\rangle \quad , \quad \varepsilon = \langle\vec{E}^2/8\pi\rangle \quad . \tag{39}$$

For gas formation by isothermal reversible charging, the volume Ω is embedded into a heat bath of temperature T. The transfer of dN charges e from infinity into the cavity Ω requires on the average the charging work $dA = d\langle U - \Psi\rangle = d\langle U\rangle$ by Eq. (36), and their thermalization at a temperature T consumes on the average the energy $dU^{th} = \frac{3}{2}KTdN$. The difference of these energies, dQ, is supplied by the heat bath. In accordance with the first law of thermodynamics

$$dQ = dU^{th} - d\langle U\rangle \tag{40}$$

since no other than electric charging work is performed on the system ($d\Omega = 0$). The associated entropy $dS = dQ/T$ is a complete differential,

$$dS = \frac{1}{T}\frac{\partial}{\partial T}(U^{th} - \langle U\rangle)dT + \frac{1}{T}\frac{\partial}{\partial\varepsilon}(U^{th} - \langle U\rangle)d\varepsilon + \frac{\partial}{\partial N}(U^{th} - \langle U\rangle)dN \quad . \tag{41}$$

Application of the condition $\partial_\varepsilon\partial_T S = \partial_T\partial_\varepsilon S$ to Eq. (41) yields the partial differential for constant N and T,

$$\partial U^{th}/\partial\varepsilon = \partial\langle U\rangle/\partial\varepsilon \quad . \tag{42}$$

Since $U^{th} = 0$ for $\varepsilon = 0$ (no thermal energy in Ω before charging), the integral of Eq. (42) is

$$U^{th} = \Omega\varepsilon \quad . \tag{43}$$

Eq. (43) could have been derived by other thermodynamic gas formation processes, e.g., by adiabatic charging of the cavity Ω. In this case $dQ = 0$, and by Eq. (40)

$$dQ = dU^{th} - d\langle U\rangle = 0 \quad : \qquad \langle U\rangle = U^{th} \quad . \tag{44}$$

Finally, $\langle U\rangle$ can also be determined as that equilibrium value which maximizes the entropy,

$$dS = T^{-1}[dU^{th} - d\langle U\rangle] = 0 \quad : \qquad \langle U\rangle = U^{th} \quad . \tag{45}$$

Eqs. (43) - (45) indicate that an equipartition between thermal energy and average microfield energy exists in statistical equilibrium. This fundamental result is explicitly (Mints 1957).

$$\frac{3}{2} NKT = \Omega\langle \vec{E}^2/8\pi\rangle \quad . \tag{46}$$

CONCLUSION

In ideal gases of charged particles, the distribution function of the collective microfields is strongly temperature and density dependent. For typical temperatures and densities of ideal systems, the r.m.s. collective microfield is by orders of magnitude larger than the quasi-static Coulomb field. In statistical equilibrium, a balance among (average) kinetic particle and collective microfield energies exists, which is independent of the process of the formation of the charged particle gas.

The derived formulas for the average microfield and energy are applicable to ideal gases consisting of one species of charged particles. Examples are electron gases in highpower tubes and diodes, and non-neutralized beams and clouds of charged particles in outer space. Our results are not applicable plasmas, since in these the electron and ion components have no independent existence (coupling of the negative and positive charges through the self-consistent field and quasi-neutral or quasi-compensated behavior).

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$n[cm^{-3}]$	$E_o[Vcm^{-1}]$	$E_W[Vcm^{-1}]$	$\overline{(\Delta E^2)}^{1/2}[Vcm^{-1}]$	γ
10^{10}	1.741×10^{0}	2.164×10^{2}	0.841×10^{2}	3.598×10^{-4}
10^{12}	3.751×10^{1}	2.164×10^{3}	0.841×10^{3}	1.670×10^{-3}
10^{14}	8.081×10^{2}	2.164×10^{4}	0.841×10^{4}	7.751×10^{-3}
10^{16}	1.741×10^{4}	2.164×10^{5}	0.841×10^{5}	3.598×10^{-2}
10^{18}	3.751×10^{5}	2.164×10^{6}	0.841×10^{6}	1.670×10^{-1}

TABLE I: E_o, E_W, $\overline{(\Delta E^2)}^{1/2}$, and γ versus n for an electron gas at $T = 10^{4}\,^{\circ}K$.

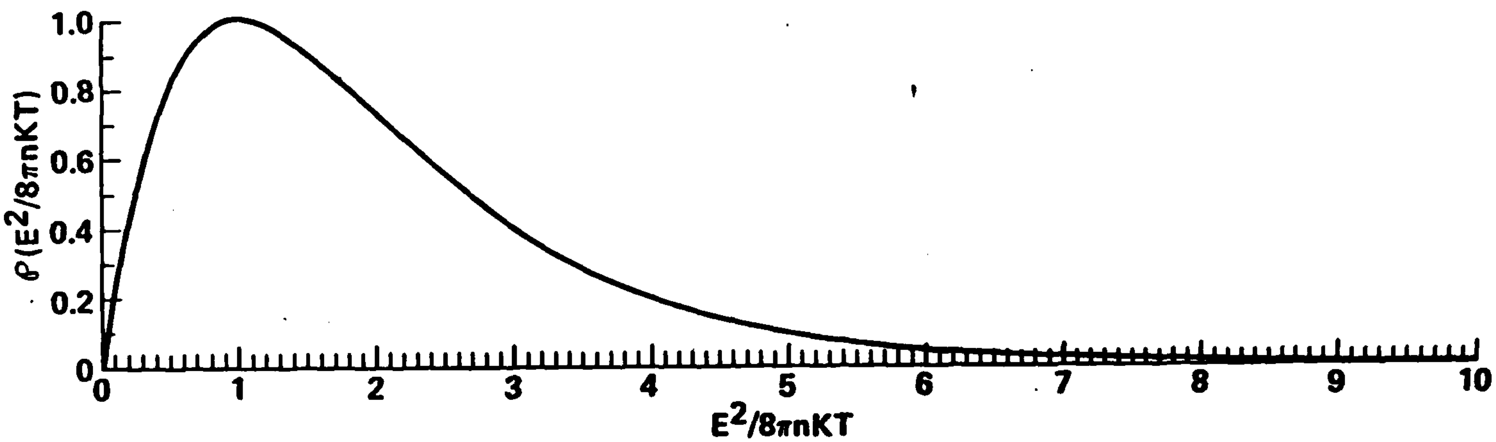

FIG. 1: Normalized probability density $P(E^2/8\pi nKT)$ versus $E^2/8\pi nKT$.

APPENDIX D

COVARIANT ELECTROMAGNETIC THEORY FOR INERTIAL FRAMES WITH SUBSTRATUM FLOW

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Abstract

Based on the Galilean relativity principle and Maxwell's equations, electromagnetic field equations are derived for inertial frames, in which the substratum of the electromagnetic waves flows with arbitrary velocity $|\vec{w}| < c$ (velocity of light). It is demonstrated that the electromagnetic field equations with electromagnetic substratum flow are strictly covariant against Galilei transformations. Wave equations, conservation and invariance theorems, and boundary conditions are derived for the electrodynamic fields in presence of electromagnetic substratum flow. Initial-boundary-value problems are solved for electromagnetic signal propagation and induction in the substratum by an integral equation method. Physical effects for the measurement of the velocity field of the electromagnetic substratum are discussed. Maxwell's conception that his equations refer to a frame of reference with resting electromagnetic substratum is confirmed, and it is shown that Maxwell's equations are also applicable to inertial frames with small substratum velocities, $|\vec{w}| << c$.

INTRODUCTION

Since the discovery of the 2.7°K cosmic microwave radiation [Wilson, 1980] Maxwell's original ideas on the propagation of electromagnetic waves in the so-called electromagnetic ether have become of renewed interest. According to Maxwell, Heaviside, Lorentz, and Poincaré the electromagnetic field equations refer to a system of reference in which the carrier of the electromagnetic waves is at rest [Whittaker, 1951]. By comparison with other wave phenomena, this restriction appears to be physically and mathematically necessary, since the Maxwell equations do not contain explicitly the velocity field w of the electromagnetic ether [Lorentz, 1909]. The most familiar "material" properties of the ether are the electric ($\varepsilon_o = 10^{-9}/36\pi$ Asec/Vm) and magnetic ($\mu_o = 4\pi\times10^{-7}$ Vsec/Am) permeabilities, the wave resistance $Z_o = (\mu_o/\varepsilon_o)^{1/2} = 376.731\ \Omega$, and the velocity of light $c_o = (\mu_o\varepsilon_o)^{-1/2} = 3\times10^8$m/sec [Stratton, 1941]. The difficulty of observing the substratum by *other than* electromagnetic experiments (e.g., measurements of the velocity of light and frequency shifts) is probably due to an extremely small interaction cross section of particles with the substratum. Evidence for this is given by Fizeau's [1851] ether drag experiment which shows that the ether is not noticeably carried along by liquid matter flowing in tubes.

As noted by cosmologists, e.g., Mansouri and Sexl [1977], "The validity of the principle of relativity (which assumes that the velocity of one and the same light signal has the same value $v_s = c_o$ in all (∞) inertial frames) seems to be less evident now than, say twenty years ago. The discovery of the cosmic background radiation has shown that cosmologically a preferred system of reference does exist." In critical analyses, Ives [1938, 1948], Builder [1958a, b], and Janossy [1953, 1963] demonstrate that the ether effects cancel out in the measurements of the velocity of light by Michelson-Morley [1887], Morley-Miller [1905], and their modern versions [Jaseja et al., 1964; Vessot et al., 1979] (using signals sent out and back). On the other hand, they show that the experiments of Sagnac [1937] and Dufour-Prunien [1937] (rotating interferometers), Ives-Stillwell [1938], and Michelson-Gale [1925] support the electromagnetic ether concept. The isotropic microwave background radiation in the universe appears to indicate thermal excitations of the ether at a nearly isotropic temperature $T_E = 2.7°K$ [Wilson, 1980] and ether velocities in the terrestrial space of the order $w \sim 10^5$m/sec [Henry, 1971].

The impossibility to carry the denial of the electromagnetic substratum in the special theory of relativity over into the general theory of relativity was clearly recognized by Einstein [1921]: "According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring rods and clocks), nor therefore any space-time in a physical sense." However, Builder [1958a,b] demonstrated for the first time, by means of quantitative theoretical arguments, that relativity theory without electromagnetic ether leads to serious physical contradictions. E.G., we now understand that *(i) the clock retardation* paradox would imply the existence of absolute velocities, which contradict the postulate of the relativity of the velocities of moving bodies, and (ii) the assumed principle of the invariance of the light velocity can only refer (if at all) to the average light velocity of a go-and-return path [Builder, 1958a, b]. Accordingly, Einstein's clock synchronization represents a "thought-ritual," which has no empirical value for the measurement of the actual velocity of light signals [Alfvén, 1977], and the special theory of relativity is a tautology based on average two-way signal velocities [Builder, 1958a,b; Janossy, 1953, 1963; Ives, 1948].

Comprehensive discussions of electromagnetic substratum physics, from the theoretical and experimental points of view, are due to Janossy [1953, 1963]. Dirac [1958] and Kaempffer [1953] introduced the ether into quantum mechanics. The non-Lorentz covariant theory of Wilson [1974] interprets elementary particles as phase changes of an ether model. Winterberg [1984] developed a nonlinear relativity theory with ether and a minimum length, which removes the singularities of quantum electrodynamics on a physical basis. It is equally remarkable that the apparent velocity dependence of mass is explainable as a particle interaction with the ether [Bagge, 1979].

The Lorentz transformations (which would hold in absence of the ether, and follow from the alleged invariance of the velocity of light signals), are not the sole transformations which leave Maxwell's equations covariant. Other (real) covariant transformations of Maxwell's equations have been found by Cunningham [1909], Bateman [1910], Fushchich [1978], and Fushchich and Nikitin [1982]. Under consideration of the quantum-mechanical commutation relations, Winterberg [1984] derived nonlinear space-time transformation for high energy systems. Typical for these transformations is the use of conventional constitutive relations $\vec{D} = \varepsilon_0\vec{E}$ and $\vec{B} = \mu_0\vec{H}$ for the free space, i.e., the vacuum is assumed to have invariant polarization properties [Einstein, 1916, 1921].

The simultaneous covariance of Maxwell's equations with respect to Lorentz and other space-time transformations suggests the existence of covariance under an over-group of these transformations [Post, 1962, 1967, 1972, 1979]. The earliest predictions of the possibility of simultaneous Lorentz-Galilei covariance of Maxwell's equations go back to Kottler [1922a,b], Cartan [1942], and van Dantzig [1934]. The over-group is identified as the nonlinear set of coordinate substitutions in space-time [Post, 1972, 1978]. The "exclusiveness" of Lorentz covariance in relativity theory is the artificial result of an (arbitrary) restriction to the unimodular choice $\sqrt{-g} = 1$ [Einstein, 1916], which eliminates other transformation groups [Post, 1972, 1978].

Kottler [1922b], Cartan [1924], van Dantzig [1934], and Post [1962] have shown that the electromagnetic field equations can be brought into a metric-free representation, i.e., Maxwell's equations exhibit a metric-independent covariance. Thus, Maxwell's equations permit a manifold of space-time transformations, if they are not restricted through the usual linear consitiutive relations $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ [Post, 1972, 1979]. Maxwell's equations are covariant even against transformations to noninertial frames, e.g. rotating frames [Post, 1967], for certain constitutive relations $\vec{D} = \vec{F}(\vec{E})$ and $\vec{B} = \vec{G}(\vec{H})$. The accepted linear constitutive equations appear to be strictly valid only for transformations between inertial frames [Post, 1972, 1979].

Maxwell's equations refer to an inertial frame Σ_0 in which the electromagnetic substratum is at rest, $\vec{w}_0 \equiv \vec{0}$, and are, for this reason, not Galilei covariant [Whittaker, 1951]. We derive herein electromagnetic field equations for inertial frames Σ with substratum velocity w from established axioms of physics (Maxwell equations and Galileian relativity principle). We demonstrate that these generalized Maxwell equations, which contain explicitly the substratum velocity $\vec{w}$, are covariant against Galilei transformations. Since these are transformations between inertial frames, the usual linear constitutive relations are assumed [Post, 1972, 1978].

The electromagnetic field equations with substratum flow $\vec{w}$ and their Galilei transformations represent a field theory for absolute or Galileian $\vec{r} = (x,y,z)$ and t coordinates. In inertial frames with substratum drift $\vec{w}$, the Coulomb fields of charged particles are deformed by convection so that a physical length contraction of material bodies in the sense of Lorentz [1909] occurs. This length contraction brings about a time dilatation since a clock can be visualized as a light signal which is reflected anisotropically in the substratum space between two mirrors held apart by a rod [Builder, 1958a, b]. For this reason, also the interrelation between the absolute Galileian and the actually measured coordinate and time differences will be discussed, which is determined by the Lorentz scaling measure $\gamma = (1 - \vec{w}^2/c^2)^{1/2}$, i.e., a constant parameter for any given inertial frame $(\vec{w})$.

The derivation of the Galilei covariant Maxwell equations for inertial frames with substratum flow are of interest for (i) mathematical and (ii) physical reasons. The generalized electromagnetic field equations provide physical foundations for investigations of the electromagnetic substratum. As applications of this theory, fundamental initial-boundary-value problems are solved analytically for electromagnetic signal propagation and induction in the substratum. We present these results for (i) theoretical discussion and (ii) comparison with experiments.

ELECTROMAGNETIC FIELD EQUATIONS WITH SUBSTRATUM

According to Maxwell, Heaviside, Lorentz, and Poincaré, the electromagnetic fields $\vec{E}^\circ(\vec{r}^\circ,t^\circ)$ and $\vec{H}^\circ(\vec{r}^\circ,t^\circ)$ in an inertial frame of reference $\sum^\circ(\vec{r}^\circ,t^\circ,\vec{w}^\circ=\vec{0})$, in which the electromagnetic ether is at rest, $\vec{w}^\circ = \vec{0}$, are determined by the classical Maxwell equations for conducting media with velocity field [Stratton, 1941]:

$$\nabla^\circ\times\vec{E}^\circ = -\partial\vec{B}^\circ/\partial t^\circ \quad , \quad (1)$$

$$\nabla^\circ\times\vec{H}^\circ = +\partial\vec{D}^\circ/\partial t^\circ + \vec{j}^\circ \quad , \quad (2)$$

$$\nabla^\circ\cdot\vec{D}^\circ = \rho^\circ \quad , \quad (3)$$

$$\nabla^\circ\cdot\vec{B}^\circ = 0 \quad , \quad (4)$$

where

$$\vec{j}^\circ = \rho^\circ\vec{v}^\circ + \sigma^\circ(\vec{E}^\circ + \vec{v}^\circ\times\vec{B}^\circ) \quad , \quad (5)$$

$$\vec{D}^\circ = \varepsilon^\circ\vec{E}^\circ \quad , \qquad \vec{B}^\circ = \mu^\circ\vec{H}^\circ \quad . \quad (6)$$

As usual, the dielectric permittivity ε°, the magnetic permeability μ°, and the electric conductivity σ° are treated as isotropic and homogeneous. Equations (1) - (6) hold not only for conducting media, but also for insulating media including the so-called "vacuum" ($\sigma^\circ \equiv 0$) of the system $\sum^\circ$, which contains electromagnetic substratum at rest, $\vec{w}^\circ = \vec{0}$. Equation (5) is Ohm's law for the current density $\vec{j}^\circ$ with space charge flow $\rho^\circ\vec{v}^\circ$.

Let Eqs. (1) - (6) be subject to a Galilei transformation of the space and time coordinates from the system $\sum^\circ(\vec{r}^\circ,t^\circ,\vec{w}^\circ=\vec{0})$ to a system $\sum(\vec{r},t,\vec{w}\neq\vec{0})$, which moves with the constant, but otherwise arbitrary, velocity $\vec{u}$ relative to $\sum^\circ$ (0° of $\sum^\circ$ and 0 of $\sum$ are assumed to coincide for $t^\circ = t = 0$, Fig. 1):

$$\vec{r} = \vec{r}^\circ - \vec{u}t^\circ \quad , \qquad t = t^\circ \quad , \quad (7)$$

$$\partial/\partial t^\circ = \partial/\partial t - \vec{u}\cdot\nabla \quad , \qquad \nabla^\circ = \nabla \quad , \quad (8)$$

with

$$\vec{E}^\circ = \vec{E} - \vec{u}\times\vec{B} \quad , \qquad \vec{H}^\circ = \vec{H} \quad , \tag{9}$$

$$\vec{j}^\circ = \vec{j} + \rho\vec{u} \quad , \qquad \rho^\circ = \rho \quad , \tag{10}$$

$$\vec{v}^\circ = \vec{v} + \vec{u} \quad , \qquad \vec{w} = -\vec{u} \quad , \tag{11}$$

as Galilei field transformations. Eqs. (8) follow from Eq. (7) by partial differentiation, $\partial f^\circ(\vec{r}^\circ,t^\circ)/\partial t^\circ = [\partial f(\vec{r},t)/\partial t]\partial t/\partial t^\circ + [\partial f(\vec{r},t)/\partial\vec{r}]\cdot\partial\vec{r}/\partial t^\circ$, and $\partial f^\circ(\vec{r}^\circ,t^\circ)/\partial\vec{r}^\circ = [\partial f(\vec{r},t)/\partial\vec{r}]\cdot\partial\vec{r}/\partial\vec{r}^\circ$, where $\partial\vec{r}/\partial t^\circ = -\vec{u}$ and $\partial\vec{r}/\partial\vec{r}^\circ = \vec{\delta}$ ($\delta_{ij} = 1$, $i = j$; $\delta_{ij} = 0$, $i \neq j$).

The Galilei field transformations (9) - (11) are established empirically, but will be justified theoretically by covariance requirements. Furthermore, since medium density and temperature are Galilei invariants,

$$\varepsilon^\circ = \varepsilon \ , \qquad \mu^\circ = \mu \ , \qquad \sigma^\circ = \sigma \quad . \tag{12}$$

The invariance of ε and μ implies the Galilei invariance of the characteristic phase speed of light [Stratton, 1941]

$$c^\circ = (\mu^\circ\varepsilon^\circ)^{-1/2} = (\mu\varepsilon)^{-1/2} = c \quad . \tag{13}$$

Since the ether is at rest, $\vec{w}^\circ = \vec{0}$, in the system $\sum^\circ$, the ether moves with the velocity $\vec{w} = -\vec{u} \neq \vec{0}$ in the system $\sum$ (Fig. 1). Application of the Galilei transformations (7) - (12) to Eqs. (1) - (6) yields, therefore, the electromagnetic field equations in the inertial frame of reference $\sum(\vec{r},t,\vec{w}\neq\vec{0})$, in which the ether streams with the velocity $\vec{w}$:

$$\nabla\times(\vec{E} + \vec{w}\times\vec{B}) = -(\partial/\partial t + \vec{w}\cdot\nabla)\vec{B} \quad , \tag{14}$$

$$\nabla\times\vec{H} = +(\partial/\partial t + \vec{w}\cdot\nabla)(\vec{D} + \varepsilon\vec{w}\times\vec{B}) + \vec{j} - \rho\vec{w} \quad , \tag{15}$$

$$\nabla\cdot(\vec{D} + \varepsilon\vec{w}\times\vec{B}) = \rho \quad , \qquad (16)$$

$$\nabla\cdot\vec{B} = 0 \quad , \qquad (17)$$

where

$$\vec{j} = \rho\vec{v} + \sigma(\vec{E} + \vec{v}\times\vec{B}) \quad , \qquad (18)$$

$$\vec{D} = \varepsilon\vec{E} \quad , \qquad \vec{B} = \mu\vec{H} \quad . \qquad (19)$$

Below, it will be demonstrated that Eqs. (14) - (19) are covariant against the three-fold infinite number of possible Galilei transformations. For this reason, Eqs. (14) - (19) are fundamental electromagnetic field equations, which hold for all inertial frames of reference $\sum(\vec{r},t,\vec{w})$, in which the ether flows with constant, but otherwise arbitrary velocity $\vec{w} \gtrless \vec{0}$. In addition, Eqs. (14) - (19) are approximately correct for inertial frames of reference $\sum(\vec{r},t,\vec{w}(\vec{r},t))$ in which the ether flow field $\vec{w}(\vec{r},t)$ is inhomogeneous, as long as the spatial and temporal nonuniformities of $\vec{w}(\vec{r},t)$ have characteristic extensions $|\Delta\vec{r}|$ in space and $|\Delta t|$ in time, which are large compared with the dimension and duration of the electromagnetic process, respectively. The latter conditions are frequently satisfied since most experiments are restricted to spatial regions which are neglegibly small compared with the universe.

At interfaces between different media, boundary conditions for the electromagnetic field vectors are required. These are obtained by integrating Eqs. (14) - (17) across the interface with normal vector $\vec{n}$. If $\vec{n}$ points from medium "1" to medium "2" and $[\vec{F}] \equiv \vec{F}_2 - \vec{F}_1$, the boundary conditions in presence of ether flow $\vec{w}$ and interface motion $\vec{v}$ are:

$$\vec{n}\times[\vec{E}] = (\vec{n}\cdot\vec{v})[\vec{B}] \quad , \qquad (20)$$

$$\vec{n}\times[\vec{H} + \vec{w}\times(\vec{D} + \varepsilon\vec{w}\times\vec{B})] = -(\vec{n}\cdot\vec{v})[\vec{D} + \varepsilon\vec{w}\times\vec{B}] + \vec{j}^* \quad , \qquad (21)$$

$$\vec{n}\cdot[\vec{D} + \varepsilon\vec{w}\times\vec{B}] = \rho^* \quad , \tag{22}$$

$$\vec{n}\cdot[\vec{B}] = 0 \quad , \tag{23}$$

where $\vec{j}^*$ is the surface current density and ρ^* is the surface charge density. In the derivation of Eqs. (20) and (21), the relations $\vec{w}\cdot\nabla\vec{B} = -\nabla\times(\vec{w}\times\vec{B})$ and $\vec{w}\cdot\nabla[\vec{D} + \varepsilon\vec{w}\times\vec{B}) - \rho\vec{w} = -\nabla\times\{\vec{w}\times(\vec{D} + \varepsilon\vec{w}\times\vec{B})\}$, both for constant $\vec{w}$, were used. The interaction of the ether with the interface is negligible, hence $[\vec{w}] = \vec{0}$. In general, the interfaces separate regions of space in which both ε, μ, and σ are different, and the interface moves with a velocity field $\vec{v}(\vec{r},t) \gtrless \vec{0}$.

The derivation of the generalized electromagnetic field equations (14) - (17) has been carried through in the MKS system for physical reasons. In comparison to the cgs system ($\vec{E}$ and $\vec{H}$ same units), the Giorgian system is superior since (i) it introduces charge as a separate independent unit and (ii) it permits to treat charge as an invariant not only for Lorentz but for all Kottler-Cartan-Dantzig-Post type space-time transformations [Post 1972, 1979] including the Galilei transformation. The invariance of charge "e" is related to the invariance of the characteristic speed of light $c_o = (\mu_o\varepsilon_o)^{-1/2}$ since $c_o \sim m^0 e^2/\varepsilon_o^1 h^1$ for dimensional reasons (no mass dependence, $m^0 = 1$). This fundamental relation appears to indicate that electromagnetic wave propagation involves quantum-mechanical interactions (h = Planck constant) of charges e associated with the ether medium (ε_o,μ_o).

GALILEI COVARIANCE OF ELECTROMAGNETIC FIELD EQUATIONS

The laws of nature are of the same form in all inertial frames [Einstein, 1916]. In view of the derivation of the electromagnetic field equations (14) - (19) with ether flow $\vec{w}$ they should be covariant against Galilei transformations. The relations for the Galilei transformation of the independent variables and the dependent fields from the inertial frame $\sum(\vec{r},t,\vec{w})$ to the inertial frame $\sum'(\vec{r}',t',\vec{w}')$, which moves with the constant, but otherwise arbitrary velocity $\vec{u}$ relative to $\sum$, are given by (0 and 0' of $\sum$ and $\sum'$ coincide for t = t' = 0, Fig. 2):

$$\vec{r}' = \vec{r} - \vec{u}t \quad , \quad t' = t \quad , \tag{24}$$

$$\partial/\partial t = \partial/\partial t' - \vec{u}\cdot\nabla' \quad , \quad \nabla = \nabla' \quad , \tag{25}$$

and

$$\vec{E} = \vec{E}' - \vec{u}\times\vec{B}' \quad , \quad \vec{H} = \vec{H}' \quad , \tag{26}$$

$$\vec{j} = \vec{j}' + \rho'\vec{u} \quad , \quad \rho = \rho' \quad , \tag{27}$$

$$\vec{v} = \vec{v}' + \vec{u} \quad , \quad \vec{w} = \vec{w}' + \vec{u} \quad , \tag{28}$$

where

$$\varepsilon = \varepsilon' \qquad \mu = \mu' \qquad \sigma = \sigma' \quad . \tag{29}$$

Substitution of Eqs. (24) - (29) into Eqs. (14) - (19) results in the electromagnetic field equations for the inertial frame $\sum'(\vec{r}', t', \vec{w}')$ with ether flow $\vec{w}'$:

$$\nabla'\times(\vec{E}' + \vec{w}'\times\vec{B}') = -(\partial/\partial t' + \vec{w}'\cdot\nabla')\vec{B}' \quad , \tag{30}$$

$$\nabla'\times\vec{H}' = +(\partial/\partial t' + \vec{w}'\cdot\nabla')(\vec{D}' + \varepsilon'\vec{w}'\times\vec{B}') + \vec{j}' - \rho'\vec{w}' \quad , \tag{31}$$

$$\nabla'\cdot(\vec{D}' + \varepsilon'\vec{w}'\times\vec{B}') = \rho' \quad , \tag{32}$$

$$\nabla'\cdot\vec{B}' = 0 \quad , \tag{33}$$

where

$$\vec{j}' = \rho'\vec{v}' + \sigma'(\vec{E}' + \vec{v}'\times\vec{B}') \quad , \qquad (34)$$

$$\vec{D}' = \epsilon'\vec{E}' \quad , \qquad \vec{B}' = \mu'\vec{H}' \quad . \qquad (35)$$

The Eqs. (30) - (35) for the inertial frame $\sum'(\vec{r}',t',\vec{w}')$ are indeed of the same form as the Eqs. (14) - (19) for the inertial frame $\sum(\vec{r},t,\vec{w})$. Thus, the Galilei covariance of the proposed electromagnetic field equations (14) - (19) with ether flow $\vec{w}$ is demonstrated. This covariance is the necessary condition for Eqs. (14) - (19) to be generally valid.

It is remarkable that the Galilei transformations (26) of the fields $\vec{E}$ and $\vec{H}$ are not symmetric. In view of Eq. (29), Eq. (22) can also be stated as

$$\vec{D} = \vec{D}' - \epsilon'\vec{u}\times\vec{B}' \quad , \qquad \vec{B} = \vec{B}' \quad . \qquad (36)$$

The analogous transformation formula for the magnetic field, $\vec{B} = \vec{B}' + \mu'\vec{u}\times\vec{D}'$, does not render Eqs. (14) - (19) Galilei covariant. This relation is incompatible because it implies magnetic charges, $\nabla\cdot\vec{B} \neq 0$, for which no experimental evidence exists.

According to Eq. (16), an ether flow $\vec{w}$ transverse to the magnetic field $\vec{B}$ induces an electric charge density ρ_w,

$$\nabla\cdot\vec{D} = \rho + \rho_w \quad , \qquad \rho_w = -\nabla\cdot(\epsilon\vec{w}\times\vec{B}) \quad . \qquad (37)$$

This interesting effect could, in principle, be used to detect the ether flow $\vec{w}$ through space charge measurements. Unfortunately, it appears that

$$\rho_w = c^{-2}\vec{w}\cdot\nabla\times\vec{H} \qquad (38)$$

is very small for laboratory experiments ($w < 10^5$ m/sec, $\nabla\times\vec{H} < 10^{10}$ A/m^2) and even for cosmic situations, e.g., for quasars ($w \leq c$, $\nabla\times\vec{H} < 10^{-4}$ A/m^2).

WAVE EQUATIONS FOR ELECTROMAGNETIC POTENTIALS

The basic Eqs. (14) - (19) represent a system of coupled partial differential equations of first order for the electromagnetic field $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$. With regard to mathematical applications, it is desirable to work with decoupled wave equations for the scalar potential $\Phi(\vec{r},t)$ and the vector potential $\vec{A}(\vec{r},t)$. From these, the electromagnetic fields are derived as partial derivatives,

$$\vec{E} = -\nabla\Phi - \partial\vec{A}/\partial t \quad , \qquad \vec{B} = \nabla\times\vec{A} \quad . \tag{39}$$

Since an arbitrary vector field $\vec{A}(\vec{r},t)$ consists of a solenoidal and an irrotational part, and $\nabla\cdot\vec{A}$ is still unspecified by Eq. (39), $\vec{A}$ is subject to the "ether gauge"

$$\nabla\cdot\vec{A} = -\mu\varepsilon(\partial/\partial t + \vec{w}\cdot\nabla)(\Phi - \vec{w}\cdot\vec{A}) \quad , \tag{40}$$

which reduces to the Lorentz gauge for $\vec{w} = \vec{0}$. The relations in Eq. (39) satisfy Eqs. (14) and (17) identically. Elimination of $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ from Eqs. (15) and (16) by means of Eq. (39) yields, under consideration of Eq. (40), the inhomogeneous wave equations for the vector potential $\vec{A}(\vec{r},t)$ and the scalar potential $\Psi(\vec{r},t) = \Phi(\vec{r},t) - \vec{w}\cdot\vec{A}(\vec{r},t)$:

$$[\mu\varepsilon(\partial/\partial t + \vec{w}\cdot\nabla)^2 - \nabla^2]\vec{A} = \mu(\vec{J} - \rho\vec{w}) \quad , \tag{41}$$

$$[\mu\varepsilon(\partial/\partial t + \vec{w}\cdot\nabla)^2 - \nabla^2](\Phi - \vec{w}\cdot\vec{A}) = \rho/\varepsilon \quad , \tag{42}$$

where $\mu\varepsilon = c^{-2}$. These hyperbolic equations exhibit the convective influence of the ether flow $\vec{w}$ on the electromagnetic potentials. Eqs. (41) and (42) reduce to the conventional wave equations for $\vec{A}(\vec{r},t)$ and $\Phi(\vec{r},t)$ in the special case of the Maxwell frame Σ° with resting ether, $\vec{w} = \vec{0}$ [Stratton, 1941].

The initial and boundary conditions for $\vec{A}(\vec{r},t)$ and $\Psi(\vec{r},t) = \Phi(\vec{r},t) - \vec{w}\cdot\vec{A}(\vec{r},t)$ follow from those for $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ via Eq. (39). Thus, the solutions $\vec{A}(\vec{r},t)$ and $\Psi(\vec{r},t)$ of the wave equations (41) and (42) can be established for known current and space charge distributions $\vec{j}(\vec{r},t)$ and $\rho(\vec{r},t)$. As an example, we give the solutions of fundamental retarded potentials of Eqs. (41) and (42), which are generated by the sources $\vec{j}(\vec{r},t)$ and $\rho(\vec{r},t)$:

$$\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\iiint R^{-1}[\vec{j}(\vec{r}^*,t-R/c) - \vec{w}\rho(\vec{r}^*,t-R/c)]d^3\vec{r}^* \tag{43}$$

$$\Phi(\vec{r},t) = \frac{1}{4\pi\varepsilon}\iiint R^{-1}[(1 - \frac{w^2}{c^2})\rho(\vec{r}^*,t-R/c) + c^{-2}\vec{w}\cdot\vec{j}(\vec{r}^*,t-R/c)]d^3\vec{r}^* \tag{44}$$

where

$$R = \sqrt{(\vec{r} - \vec{r}^*)^2} \quad , \quad R(\vec{r},\vec{r}^*) = R'(\vec{r}',\vec{r}^{*\prime}) \quad , \tag{45}$$

is the distance between the field point $(\vec{r})$ and the source point $(\vec{r}^*)$. By Eqs. (43) and (44), the sources $\vec{j}(\vec{r}^*,\tau)$ and $\rho(\vec{r}^*,\tau)$ at the source point $\vec{r}^*$ contribute, at the retarded time $\tau = t - R/c$, to the fields $\vec{A}(\vec{r},t)$ and $\Phi(\vec{r},t)$ at the point $\vec{r}$ and time t of observation. Since $R(\vec{r},\vec{r}^*)$ and $c = (\mu\varepsilon)^{-1/2}$ are Galilei invariants by Eqs. (24) and (29), the time retardation $\Delta t = -R/c$ is a Galilei invariant as expected [$\Delta t = \Delta t'$ by Eq. (24)].

The retarded potential solutions (43) and (44) refer to the inertial frame $\sum(\vec{r},t,\vec{w})$, in which the ether velocity is $\vec{w} \neq \vec{0}$. They indicate that the corresponding solutions for the Maxwell frame $\sum^\circ(\vec{r}^\circ,t^\circ,\vec{w}^\circ = \vec{0})$ are approximately valid also for $|\vec{w}^\circ| \ll c$. However, for $|\vec{w}| \to c$, the effects of the ether flow $\vec{w}$ on the potentials $\vec{A}(\vec{r},t)$ and $\Phi(\vec{r},t)$ are quantitatively significant. Note that also the nature of the $\Phi(\vec{r},t)$ solution changes with respect to its sources as $|\vec{w}|$ increases from 0 towards c.

The ether gauge (40) and the wave equations (41) and (42) are covariant against the Galilei transformations (24) - (29). This covariance is obvious from the co-/ invariance of the operators and field expressions in Eqs. (40) - (42), namely:

$$\partial/\partial t + \vec{w}\cdot\nabla = \partial/\partial t' + \vec{w}'\cdot\nabla' \quad , \qquad \nabla = \nabla' \quad , \tag{46}$$

$$\vec{A} = \vec{A}' \quad , \tag{47}$$

$$\Phi - \vec{w}\cdot\vec{A} = \Phi' - \vec{w}'\cdot\vec{A}' \quad , \tag{48}$$

$$\vec{j} - \rho\vec{w} = \vec{j}' - \rho'\vec{w}' \quad , \tag{49}$$

$$\rho = \rho' \quad , \tag{50}$$

and Eq. (29). The unprimed and primed fields refer to the inertial frames $\sum(\vec{r},t,\vec{w})$ and $\sum'(\vec{r}',t',\vec{w}')$, respectively. Since the relative system velocity is $\vec{u} = \vec{w} - \vec{w}' = \vec{v} - \vec{v}'$ (Fig. 2), Eqs. (46), (48), and (49) imply the Galilei co-/ invariants

$$\partial/\partial t + \vec{v}\cdot\nabla = \partial/\partial t' + \vec{v}'\cdot\nabla' \quad , \tag{51}$$

$$\Phi - \vec{v}\cdot\vec{A} = \Phi' - \vec{v}'\cdot\vec{A}' \quad , \tag{52}$$

$$\vec{j} - \rho\vec{v} = \vec{j}' - \rho'\vec{v}' \quad . \tag{53}$$

In terms of the electromagnetic fields, the Galilei invariants (47), (48), and (52) assume by Eq. (39) the form [Stratton, 1941].

$$\vec{B} = \vec{B}' \quad , \tag{54}$$

$$\vec{E} + \vec{w}\times\vec{B} = \vec{E}' + \vec{w}'\times\vec{B}' \quad , \tag{55}$$

$$\vec{E} + \vec{v}\times\vec{B} = \vec{E}' + \vec{v}'\times\vec{B}' \quad . \tag{56}$$

By means of the invariants (46), (49), (50), and (53) - (56), the Galilei covariance of the basic electromagnetic field equations (14) - (19) with ether flow $\vec{w}$ is now immediately recognized.

In the special case $\vec{w} = \vec{0}$, the new wave equations (41) and (42) with ether flow combine to the relativistic wave equation $\Box\{\vec{A}, i\Phi/c\} = -\mu\{\vec{j}, ic\rho\}$ with the Lorentz covariant operator $\Box = \nabla^2 - c^{-2}\partial_t^2$. This D'Alembertian can, in no way whatever, describe anisotropic light propagation or the nonreciprocal asymmetry between the clockwise and counterclockwise beams observed in the Sagnac experiment [Post, 1967]. Anisotropy and nonreciprocity require mixed space-time derivatives $\nabla_i\partial_t$ in the wave equation [Post, 1967]. These are generated by the ether convection $\vec{w}\cdot\nabla$ in the proposed wave equations (41) and (42) with the nonsymmetric operator $\tilde{\Box} = \nabla^2 - c^{-2}(\partial_t + \vec{w}\cdot\nabla)^2$. The space-time symmetry is destroyed by the ether flow $\vec{w}$, i.e., exists only in the ether rest frame where $\vec{w} \equiv \vec{0}$.

ELECTRODYNAMIC CONSERVATION THEOREMS

In the generalized electrodynamics with ether flow $\vec{w}$, charge, energy, and momentum of the fields are conserved in all inertial frames $\sum(\vec{r},t,\vec{w})$ with ether motion $\vec{w}$. These conservation theorems are presented and shown to be Galilei covariant.

Charge Conservation. The divergence of Eq. (15) yields, under consideration of the vector identity for arbitrary fields $\vec{a}(\vec{r},t)$,

$$\nabla\cdot[(\partial/\partial t + \vec{w}\cdot\nabla)\vec{a}] = (\partial/\partial t + \vec{w}\cdot\nabla)\nabla\cdot\vec{a} \quad , \qquad (57)$$

and Eq. (16), the conservation equation for the charge density in the inertial frame $\sum(\vec{r},t,\vec{w})$:

$$(\partial/\partial t + \vec{w}\cdot\nabla)\rho = -\nabla\cdot(\vec{j} - \rho\vec{w}) \quad . \qquad (58)$$

This equation is Galilei covariant in view of Eqs. (46), (49), and (50). Eq. (58) is equivalent to the usual charge continuity equation [Stratton, 1941]

$$\partial\rho/\partial t = -\nabla\cdot\vec{j} \quad . \qquad (59)$$

Electromagnetic Energy Conservation. In accordance with the vector relations for arbitrary fields $\vec{a}(\vec{r},t)$ and $\vec{b}(\vec{r},t)$,

$$\nabla\cdot(\vec{a}\times\vec{b}) = \vec{b}\cdot\nabla\times\vec{a} - \vec{a}\cdot\nabla\times\vec{b} \quad , \qquad (60)$$

$$\vec{a}\cdot(\partial/\partial t + \vec{w}\cdot\nabla)\vec{a} = (\partial/\partial t + \vec{w}\cdot\nabla)\vec{a}^2/2 \quad , \qquad (61)$$

scalar multiplications of (i) Eq. (14) by $\vec{H}$ and (ii) Eq. (15) by $(\vec{E} + \vec{w}\times\vec{B})$, - and subtraction of the resulting equations (i) and (ii) - , results in the conservation equation for the field energy in the inertial frame $\sum(\vec{r},t,\vec{w})$:

$$(\partial/\partial t + \vec{w}\cdot\nabla)[\frac{\varepsilon}{2}(\vec{E} + \vec{w}\times\vec{B})^2 + \frac{\mu}{2}\vec{H}^2] = -\nabla\cdot[(\vec{E} + \vec{w}\times\vec{B})\times\vec{H}] - (\vec{J} - \rho\vec{w})\cdot(\vec{E} + \vec{w}\times\vec{B}) \quad . \tag{62}$$

This equation is Galilei covariant by Eqs. (46), (49), (54), and (55). Equation (62) can be rewritten in the form

$$(\partial/\partial t + \vec{w}\cdot\nabla)(U_e + U_m) = -\nabla\cdot\vec{P} - Q \tag{63}$$

where

$$U_e = \frac{1}{2}\varepsilon\,(\vec{E} + \vec{w}\times\vec{B})^2 \quad , \tag{64}$$

$$U_m = \frac{1}{2}\mu\,\vec{H}^2 \quad , \tag{65}$$

$$\vec{P} = (\vec{E} + \vec{w}\times\vec{B})\times\vec{H} \quad , \tag{66}$$

$$Q = (\vec{J} - \rho\vec{w})\cdot(\vec{E} + \vec{w}\times\vec{B}) \quad , \tag{67}$$

are the electric and magnetic energy densities, the Poynting vector, and the ohmic power density in presence of ether flow $\vec{w}$. Note that Eqs. (64) - (67) define generalized concepts for energy density, energy flux, and power density of the electromagnetic field, which are Galilei invariant.

Electromagnetic Momentum Conservation. The generalized electric and magnetic stress tensors $\overleftrightarrow{T}_{e,m}$ in presence of ether flow $\vec{w}$ are introduced by means of the vector identity

$$(\nabla\times\vec{a})\times\vec{a} = \nabla\cdot(\vec{a}\vec{a} - \frac{1}{2}\vec{a}^2\overleftrightarrow{\delta}) - \vec{a}\nabla\cdot\vec{a} \quad , \tag{68}$$

which gives

$$\varepsilon[\nabla\times(\vec{E}+\vec{w}\times\vec{B})]\times(\vec{E} + \vec{w}\times\vec{B}) = \nabla\cdot\overleftrightarrow{T}_e - \varepsilon(\vec{E} + \vec{w}\times\vec{B})\nabla\cdot(\vec{E} + \vec{w}\times\vec{B}) \quad , \tag{69}$$

$$\mu^{-1}(\nabla\times\vec{B})\times\vec{B} = \nabla\cdot\overleftrightarrow{T}_m - \mu^{-1}\vec{B}\nabla\cdot\vec{B} \quad , \tag{70}$$

where

$$\overleftrightarrow{T}_e = \varepsilon(\vec{E} + \vec{w}\times\vec{B})(\vec{E} + \vec{w}\times\vec{B}) - \frac{1}{2}\varepsilon(\vec{E} + \vec{w}\times\vec{B})^2\overleftrightarrow{\delta} \quad , \tag{71}$$

$$\overleftrightarrow{T}_m = \mu^{-1}\vec{B}\vec{B} - \frac{1}{2}\mu^{-1}\vec{B}^2\overleftrightarrow{\delta} \quad . \tag{72}$$

The dyadics (71) and (72) are Galilei invariants by Eqs. (54) and (55). Vectorial multiplications of (i) Eq. (14) by $\times\varepsilon(\vec{E} + \vec{w}\times\vec{B})$ and (ii) Eq. (15) by $\times\vec{B}$, and addition of the resulting equations (i) and (ii), gives the conservation equation for the electromagnetic momentum in the inertial frame $\sum(\vec{r},t,\vec{w})$:

$$(\partial/\partial t + \vec{w}\cdot\nabla)[\mu\varepsilon(\vec{E} + \vec{w}\times\vec{B})\times\vec{H}] = \nabla\cdot(\overleftrightarrow{T}_e + \overleftrightarrow{T}_m) - \rho(\vec{E} + \vec{w}\times\vec{B}) - (\vec{j} - \rho\vec{w})\times\vec{B} \ , \quad (73)$$

or

$$(\partial/\partial t + \vec{w}\cdot\nabla)(\mu\varepsilon\vec{P}) = \nabla\cdot(\overleftrightarrow{T}_e + \overleftrightarrow{T}_m) - \rho\vec{E} - \vec{j}\times\vec{B} \quad . \quad (74)$$

These equations are Galilei covariant by Eqs. (46), (49), (50), (55), (71), and (72). Equation (73) or (74) is a relation through which the forces on charge $\rho(\vec{r},t)$ and current $\vec{j}(\vec{r},t)$ densities can be expressed by the electromagnetic fields $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ in the medium, which is assumed to be homogeneous with respect to ε, μ, and σ. In particular, if the electromagnetic momentum change $(\partial_t + \vec{w}\cdot\nabla)(\mu\varepsilon\vec{P})$ is negligible, the electromagnetic force density equals the divergence of the field stress tensor, $\rho\vec{E} + \vec{j}\times\vec{B} \cong \nabla\cdot(\overleftrightarrow{T}_e + \overleftrightarrow{T}_m)$ [Stratton, 1941].

Within the frame of the Galilei covariant electrodynamics for inertial systems $\sum(\vec{r},t,\vec{w})$ with ether flow $\vec{w}$, generalized conservation theorems for energy [Eq. (67)] and momentum [Eq. (73)] of the electromagnetic field have been found. These reduce exactly and approximately to the ordinary electromagnetic conservation equations for ether velocities $\vec{w} = \vec{0}$ and $|\vec{w}| \ll c$, respectively. Equations (67) and (73) predict significant physical effects for large ether velocities $|\vec{w}| \to c$, e.g., in the vicinity of distant galaxies and quasars with extremely large ether expansion drifts. A verification of the ether terms in Eqs. (67) and (73) by means of laboratory experiments may be difficult, since macroscopic platforms can presently not be accelerated to speeds $v > 10^4$ m/sec.

SIGNAL PROPAGATION IN SUBSTRATUM

According to the solutions of Maxwell's equations (1) - (6) for the inertial frame of the resting ether ($\vec{w}^\circ = \vec{0}$), light signals propagate with an isotropic speed $v_s^\circ = c = (\mu\varepsilon)^{-1/2}$, which is independent of the velocity and acceleration of the source. This independence from velocity and acceleration of the emitter holds also for signal propagation described by the Galilei covariant electromagnetic field equations (14) - (19). However, the signal velocity $\vec{v}_s = \vec{v}_s(\vec{r},\vec{w})$ is no longer isotropic in inertial frames $\sum(\vec{r},t,\vec{w})$ with ether flow $\vec{w} \neq \vec{0}$.

In order to understand signal propagation in the ether, consider the elementary excitation of an electromagnetic wave pulse by the sudden application of a current pulse $\vec{J}^*(t)$[A/m] to the surface x = 0 of an ideal conductor ($\sigma \to \infty$) at time t = 0,

$$\vec{J}^*(t) = \{0,0,\hat{J}H(t)\} \quad , \quad x = 0 \quad , \quad t \geqslant 0 \quad , \tag{75}$$

where H(t) is the Heaviside step distribution, $dH(t)/dt = \delta(t)$. The resulting electromagnetic wave emitted from the "sheet antenna" at x = 0 at time t = 0 is of the form (Fig. 3)

$$\vec{B} = \{0,B(x,t),0\} \quad , \quad \vec{E} = \{0,0,E(x,t)\} \quad . \tag{76}$$

The propagation of the wave (76) in the charge ($\rho = 0$) and current ($\vec{J} = 0$) free space x > 0 with ether flow parallel to the wave propagation (Fig. 3)

$$\vec{w} = \{w_{\|},0,0\} \quad , \quad w_{\|} \gtrless 0 \quad , \tag{77}$$

is determined by the hyperbolic initial-boundary-value problem [derived from Eqs. (14) - (17) and Eqs. (20) - (23)]:

$$(\partial/\partial t + w_{\parallel}\partial/\partial x)^2 B = c^2\partial^2 B/\partial x^2 \quad , \quad 0 < x < \infty \quad , \tag{78}$$

$$B(x,t=0) = 0 \quad , \quad 0 < x < \infty \quad , \tag{79}$$

$$\partial B(x,t=0)/\partial t = 0 \quad , \quad 0 < x < \infty \quad , \tag{80}$$

$$B(x=0,t) = \mu(1 - w_{\parallel}^2/c^2)^{-1} j^*(t) \quad , \quad 0 < t < \infty \quad . \tag{81}$$

The Laplace transform method gives the solution of Eqs. (78) - (81) as the integral functional

$$B(x,t) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} F(s)e^{ts-(x-w_{\parallel}t)s/c}ds \tag{82}$$

where

$$\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} F(s)e^{(1+w_{\parallel}/c)ts}ds = \mu(1 - w_{\parallel}^2/c^2)^{-1} j^*(t) \tag{83}$$

by the boundary condition (81). The integral equation (83) for the Laplace amplitude F(s) has closed-form solutions for certain $j^*(t)$ functions, e.g.,

$$F(s) = \mu(1 - w_{\parallel}^2/c^2)^{-1}\hat{J}s^{-1} \quad , \quad |w_{\parallel}| < c \quad , \tag{84}$$

for the boundary value (75). Equations (82) and (84) combine to the wave pulse solution:

$$B(x,t) = \mu(1 - w_{\parallel}^2/c^2)^{-1}\hat{J}H(t - \frac{x}{w_{\parallel}+c}) \quad , \quad x > 0 \quad , \tag{85}$$

with

$$E(x,t) = -(w_{\parallel}+c)\mu(1 - w_{\parallel}^2/c^2)^{-1}\hat{J}H(t - \frac{x}{w_{\parallel}+c}) \quad , \quad x > 0 \quad , \tag{86}$$

by Eq. (14). Equations (85) and (86) follow from the discontinuous integral [Eqs. (82) and (84)],

$$\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} s^{-1}e^{ts-(x-w_{\parallel}t)s/c}ds = H(t - \frac{x-w_{\parallel}t}{c}) \tag{87}$$

where

$$H\left(t - \frac{x - w_{\parallel} t}{c}\right) = H\left(t - \frac{x}{w_{\parallel} + c}\right) \quad , \quad |w_{\parallel}| < c \quad . \tag{88}$$

The results (85) and (86) are physically interesting in several respects. As typical for hyperbolic equations or processes with finite characteristic speed $c < \infty$, the wave fields are discontinuous at the wave front (Fig. 3). The position and speed of the signal front are in the inertial frame $\sum(x,y,z,t,w_{\parallel})$

$$\xi(t) = (w_{\parallel} + c)t \gtrless ct \quad , \quad w_{\parallel} \gtrless 0 \quad , \tag{89}$$

$$v_s^{\parallel} = w_{\parallel} + c \gtrless c \quad , \quad w_{\parallel} \gtrless 0 \quad . \tag{90}$$

Accordingly, the electromagnetic signal propagates with a velocity $v_s^{\parallel} \gtrless c$ depending on whether the propagation occurs in (>) or opposit (<) to the direction of the ether flow $\vec{w}_{\parallel}$. Only in inertial frames moving with the ether, the signal propagates with the velocity $v_s^{\circ} = c$ in all directions, i.e., the Galilei invariant $c = (\mu\varepsilon)^{-1/2}$ is the speed of light relative to the ether. Apparently, c represents the upper limit for the ether velocity, $|\vec{w}| < c$, since $|\vec{B}| \rightarrow \infty$ and $|\vec{E}| \rightarrow \infty$ for $|\vec{w}| \rightarrow c$ by Eqs. (85) and (86). It is also seen that for the electric and magnetic field intensities of wave phenomena

$$|\vec{E}| = |w_{\parallel} + c| \cdot |\vec{B}| \quad , \quad |w_{\parallel} + c| \gtrless c \quad , \quad w_{\parallel} \gtrless 0 \quad . \tag{91}$$

This relation could possibly be used for the determination of the ether speed $|w_{\parallel}|$ through very accurate interferometric comparisons of the electric and magnetic amplitudes of waves.

In the same way, the initial-boundary-value problems for signals propagating in directions perpendicular to the ether velocity $\vec{w}$ can be solved, with $v_s^{\perp} = (c^2 - w^2)^{1/2}$. Accordingly, electromagnetic signal propagation is anisotropic $v_s^{\parallel} \neq v_s^{\perp}$ in inertial frames with ether flow.

ELECTROMAGNETIC INDUCTION IN SUBSTRATUM

As an elementary model for the generation of electromagnetic pulses by induction in the ether, consider a plane copper slab (initially at $x = 0$), which is accelerated at time $t = 0$ across a homogeneous magnetic field $\vec{B}_o = \{0, B_o, 0\}$ to a velocity $\vec{v} = \{\dot{a}(t), 0, 0\}$, so that the position of the front surface is at $x = a(t)$ at time t with $a(t=0) = 0$ (Fig. 4). The duration Δt of the (explosion driven) piston motion is assumed to be small compared with the field diffusion time $t_D = \mu\sigma d^2$ (slab thickness d). The motion of the quasi-ideal conductor ($\sigma \to \infty$) across $\vec{B}_o$ induces at its front surface an electromagnetic field of the form

$$\vec{B} = \{0, -\partial A(x,t)/\partial x, 0\} \quad , \quad \vec{E} = \{0, 0, -\partial A(x,t)/\partial t\} \tag{92}$$

which propagates into the space $x > a(t)$ of Fig. 4. Let the ether flow be parallel to the direction of wave propagation, $\vec{w} = \{w_\parallel, 0, 0\}$, $w_\parallel \gtrless 0$. The vector potential $\vec{A} = \{0, 0, A(x,t)\}$ is then determined by the hyperbolic initial-boundary-value problem for the homogeneous wave equation (41):

$$(\partial/\partial t + w_\parallel \partial/\partial x)^2 A = c^2 \partial^2 A/\partial x^2 \quad , \quad a(t) < x < \infty \quad , \tag{93}$$

$$A(x, t=0) = -B_o x \quad , \quad 0 < x < \infty \quad , \tag{94}$$

$$\partial A(x, t=0)/\partial t = 0 \quad , \quad 0 < x < \infty \quad , \tag{95}$$

$$[\partial A(x,t)/\partial t + \dot{a}(t) \partial A(x,t)/\partial x]_{x=a(t)} = 0 \quad , \quad 0 < t < \infty \quad . \tag{96}$$

Equations (94) - (96) consider that $B(x,t=0) = B_o$, $E(x,t=0) = 0$, and $E(x,t) + v(t)B(x,t) = 0$ at the moving piston ($\sigma = \infty$) surface $x = a(t)$. The solution of Eqs. (93) - (96) is by the Laplace transform method

$$A(x,t) = -B_o x + \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} G(s)e^{ts-(x-w_\parallel t)s/c}\,ds \tag{97}$$

where

$$\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} sG(s)e^{ts-[a(t)-w_\parallel t]s/c}\,ds = \dot{a}(t)B_o/[1 - \frac{\dot{a}(t) - w_\parallel}{c}] \tag{98}$$

by the boundary condition (96). The integral equation (98) for the Laplace amplitude $G(s)$ has closed-form solutions for certain piston motions $\dot{a}(t)$, e.g., for the Heaviside velocity pulse

$$\dot{a}(t) = v_o H(t) \quad , \quad G(s) = v_o B_o s^{-2}/(1 - \frac{v_o - w_\parallel}{c}) \quad , \quad |w_\parallel| < c \quad . \tag{99}$$

From Eqs. (97) and (99) result, under consideration of Eq. (92), the solution for the induced electromagnetic pulse fields:

$$A(x,t) = -B_o x + \frac{(1+w_\parallel/c)}{(1+w_\parallel/c - v_o/c)} v_o B_o (t - \frac{x}{w_\parallel + c})H(t - \frac{x}{w_\parallel + c}) \quad , \quad x > v_o t \, ,$$

$$= 0 \qquad , \quad x < v_o t \, , \tag{100}$$

$$B(x,t) = B_o + \frac{v_o/c}{(1+w_\parallel/c - v_o/c)} B_o H(t - \frac{x}{w_\parallel + c}) \quad , \quad x > v_o t \, ,$$

$$= 0 \qquad , \quad x < v_o t \, , \tag{101}$$

$$E(x,t) = -\frac{v_o/c}{(1+w_\parallel/c - v_o/c)} (w_\parallel + c)B_o H(t - \frac{x}{w_\parallel + c}) \quad , \quad x > v_o t \, ,$$

$$= 0 \qquad , \quad x < v_o t \, . \tag{102}$$

In the derivation of Eqs. (100) - (102), the discontinuous Laplace integrals have been expressed as Heaviside impulse functions, in accordance with Eqs. (87) and (88).

Equations (101) and (102) indicate that the electromagnetic wave pulse induced at the front surface $x = v_o t$ of the conductor, which pushes the flux of the magnetic field $\vec{B}_o$, is discontinuous at the wave front $\xi(t) = (w_\parallel + c)t \gtrsim ct$

for $w_{\|} \gtrless 0$, which propagates with the speed $v_S^{\|} = w_{\|} + c \gtrless c$ for $w_{\|} \gtrless 0$ in the inertial frame $\sum(x,y,z,t,w_{\|})$. Figure 4 shows the induced magnetic field pulse $\tilde{B}(x,t) = B(x,t) - B_o$, which occupies the space $v_o t < x < (w_{\|} + c)t$ at time t. Thus, the electromagnetic signal propagates with a speed $v_S^{\|} \gtrless c$ in the inertial frame $\sum$ depending on whether the propagation occurs in (>) or opposite (<) to the direction of ether flow $\vec{w}$. The electric and magnetic wave intensities are interrelated by $|E| = |w_{\|} + c| \cdot |\tilde{B}|$. Equations (101) and (102) indicate that for ether drifts $\vec{w}_{\|}$ opposite to the piston velocity $\vec{v}_o$

$$|B(x,t) - B_o| \to \infty \quad \text{for} \quad v_o - w_{\|} \to c \quad , \quad w_{\|} < 0 \quad , \tag{103}$$

$$|E(x,t)| \to \infty \quad \text{for} \quad v_o - w_{\|} \to c \quad , \quad w_{\|} < 0 \quad . \tag{104}$$

Accordingly, copious amounts of radiation would be produced by magnetic flux (B_o) pushers in regions of space with large ether drifts $10^{-1}c < |w_{\|}| < c$ (distant galaxies, quasars). On principle, even the limit $v_o - w_{\|} = c$ is achievable if $-w_{\|}$ deviates from c by not more than the conductor speed v_o. It is remarkable that the limit $v_o - w_{\|} = c$ is not even approximately realizable in absence of ether flow ($w_{\|} = 0$) since $v_o << c$ for macroscopic bodies.

DISPERSION AND FREQUENCY SHIFT IN SUBSTRATUM

In a homogeneous dielectric (ε,μ) or the free ether space (ε_o,μ_o) of the Maxwell frame $\Sigma^\circ(\vec{r}^\circ,t^\circ,\vec{w}^\circ = \vec{0})$ in which the ether is at rest, the frequency and wavelength of electromagnetic waves are interrelated by $\omega = |\vec{k}|c$ where $|\vec{k}| = 2\pi/\lambda$. In an inertial frame $\Sigma(\vec{r},t,\vec{w})$ with ether flow $\vec{w}$, the dispersion of monochromatic waves of frequency ω

$$\vec{A}_k(\vec{r},t) = Re\vec{A}_k e^{i(\vec{k}\cdot\vec{r}-\omega t)} \tag{105}$$

in absence of space charges $(\rho = 0)$ and electric currents $(\vec{j} = \vec{0})$ is determined by the homogeneous wave equation (41) for the vector potential with ether flow $\vec{w}$,

$$(\partial/\partial t + \vec{w}\cdot\nabla)^2\vec{A} = c^2\nabla^2\vec{A} \quad . \tag{106}$$

Substitution of Eq. (105) into Eq. (106) yields, since $\vec{A}_k \neq \vec{0}$ (condition for nontrivial solution), the dispersion equation $\omega = \omega(\vec{k})$ for electromagnetic waves in the ether:

$$\omega = |\vec{k}|c + \vec{k}\cdot\vec{w} \quad , \qquad c = (\mu\varepsilon)^{-1/2} \quad . \tag{107}$$

Thus, in presence of ether flow $\vec{w}$, electromagnetic waves exhibit in the inertial frame Σ a frequency shift

$$\Delta\omega = \vec{k}\cdot\vec{w} \gtrless 0 \quad , \qquad \cos(\vec{k},\vec{w}) \gtrless 0 \quad , \tag{108}$$

which vanishes for propagation $\vec{k}$ perpendicular to the ether flow $\vec{w}$. Equation (108) predicts a red-shift $(\Delta\omega < 0)$ or a blue-shift $(\Delta\omega > 0)$ for electromagnetic waves propagating with a wave vector component $\vec{k}_\parallel$ which is antiparallel or parallel to the ether velocity $\vec{w}$. The physical reason for this frequency shift is the propagation of waves and signals with the phase velocity

$$v_p = c + (\vec{k}\cdot\vec{w})/|\vec{k}| \quad . \tag{109}$$

in the inertial frame $\sum$, i.e., the propagation of electromagnetic waves with the velocity c relative to the moving ether. For laboratory and terrestrial applications, the frequency shift is in general small

$$|\Delta\omega| << \omega \quad \text{for} \quad |\vec{w}| << c \quad . \tag{110}$$

The red-shifts of spectral lines observed in distant galaxies have been the subject of different explanations, with the Doppler red-shift due to the rapid recession of the galaxies [Humason, 1956] being now widely accepted. The longitudinal Doppler red-shift $\Delta\omega_D = -|\vec{k}|\cdot|\vec{v}_Q|$, where $\vec{v}_Q$ is the velocity of the light source (galaxy), is of the same order-of-magnitude as the longitudinal ether red-shift from Eq. (108), $\Delta\omega = -|\vec{k}|\cdot|\vec{w}|$, if $w \doteq v_Q$. Since the ether probably "expands" as the masses of the universe recede (relative to the earth), both the Doppler and ether red-shifts have to be considered in the evaluation of the velocity $v_Q \doteq w$ of the galaxies from the experimental red-shift data[30] $c\Delta\lambda/\lambda$. These indicate ether drift velocities $w \sim 10^{-1}c$ to $4\times10^{-1}c$ [Humason, 1956] The ether red-shift represents, therefore, another physical effect for the experimental investigation of the substratum.

The presented applications provide an impression of the electromagnetic phenomena which can be expected in presence of ether flow. The theory has significant other applications in connection with the interaction of radiation and charged particles with the ether under laboratory conditions.

GALILEAN AND MEASURED COORDINATES

The generalized electromagnetic field equations (14)-(19) with substratum and their Galilei transformation (24)-(28) represent a field theory in terms of absolute or Galilean space $r = (x,y,z)$ and time t coordinates, i.e., the Galilean coordinate differences are the same in all inertial frames, $\Delta\vec{r} = \vec{inv}$ and $\Delta t = inv$. According to Lorentz [1909], a measuring rod resting parallel to the ether velocity $\vec{w}$ in an inertial frame $\Sigma(r,t,\vec{w})$ has there the reduced length $L(\vec{w}) = L_o(1 - \vec{w}^2/c^2)^{1/2}$ due to the flattening of its microscopic Coulomb fields by the ether flow (L_o = proper rod length in the ether rest frame Σ_o). Recognizing that a clock can be visualized as a system reflecting a light signal back and forth between two mirrors held apart by a rod, Builder [1958 a,b] demonstrated that the period of a clock is increased to $T(\vec{w}) = T_o/(1 - \vec{w}^2/c^2)^{1/2}$ in the inertial frame $\Sigma(r,t,\vec{w})$ with ether flow $\vec{w}$, as the combined result of the rod contraction and the anisotropy of light propagation between the mirrors (T_o = proper clock period in the ether rest frame Σ_o). For these physical reasons, the differences Δ of the absolute Galilean coordinates r,t of the inertial frame $\Sigma(r,t,\vec{w})$ are related to the measured (m) space and time coordinate differences in this reference frame by ($\parallel$ and $\perp$ to w)

$$\Delta\vec{r}_m^{\parallel} = \gamma\Delta\vec{r}^{\parallel} \quad , \quad \Delta\vec{r}_m^{\perp} = \Delta\vec{r}^{\perp} \quad , \quad \Delta t_m = \Delta t/\gamma \quad , \tag{111}$$

where

$$\gamma = (1 - \vec{w}^2/c^2)^{1/2} \quad , \quad 0 \leq |\vec{w}| < c \quad . \tag{112}$$

The measured coordinates $\vec{r}_m^{\parallel}$ and t_m vary in accordance with the $\gamma(\vec{w})$ of the respective inertial frame Σ. For mathematical ($\gamma^{-1} < \infty$) and physical (violation of causality principle) reasons, the ether speeds are restricted to values $0 \leq |\vec{w}| < c$ (in agreement with observation). Since the scaling factor $\gamma(\vec{w})$ is a constant for a given inertial frame $\Sigma(r,t,\vec{w})$, the calculation of the measured

(m) coordinates from the Galilean coordinates, and vice versa, by means of Eqs. (111) - (112) is elementary in applications.

In order to illustrate the use of the Galilean space and time coordinates, the Doppler effect shall be analyzed (within the frame of the presented theory) for an emitter A and a receiver B, which move along the x-axis with the velocities $u \gtrless 0$ and $v \gtrless 0$ relative to the ether frame Σ_o, respectively. The Galilean position coordinates of A and B are at the absolute time $t \geq 0$ of the ether frame Σ_o

$$x_A(t) = a + ut , \quad x_B(t) = b + vt , \tag{113}$$

where a and b are the initial positions. The Galilean time t is counted by similar (synchronized) clocks, distributed over the ether frame Σ_o (isotropic light propagation). The time periods of the emitter and receiver shall be the same, $T_A = T = T_B$, when A and B are at rest in Σ_o. Let A emit signals with period T_A at the times

$$t_n = t_o + nT_A , \quad n = 0,1,2, \cdots . \tag{114}$$

At a time $t > t_n$, the n-th signal emitted by A reaches the location on the x-axis

$$x_n(t) = x_A(t_n) + c(t - t_n) = a - (c - u)t_n + ct. \tag{115}$$

Accordingly, the n-th signal of the emitter A will hit the receiver B at a time t_n^* determined by $x_n(t=t_n^*) = x_B(t=t_n^*)$, or

$$a - (c - u)t_n + ct_n^* = b + vt_n^* . \tag{116}$$

Hence

$$t_n^* = t_o^* + nT_A^* , \quad n = 0,1,2, \cdots , \tag{117}$$

where

$$t_o^* \equiv t_o + (b - a)/(c - v) \quad , \tag{118}$$

$$T_A^* = T_A(c - u)/(c - v) \quad . \tag{119}$$

Equation (119) indicates that the period T_A of the emitter A is "observed" as a period T_A^* at the location of the receiver B. So far, the considerations have been purely "classical" or "Galilean."

In experiments, one compares the period T_m^B (measured at B) of the receiver B with the period T_{Am}^* (measured at B, too) of the emitter A. According to Eqs. (111) and (112)

$$T_{Am}^* = T_A^*/(1 - u^2/c^2)^{1/2} \ , \quad T_{Bm} = T_B/(1 - v^2/c^2)^{1/2} \tag{120}$$

since A moves with the volocity u and B moves with the velocity v relative to the ether, $\sum_o$. By Eqs. (119) - (120), the measured period ratio is ($T_A = T_B$)

$$T_{Bm}/T_{Am}^* = [(c + u)(c - v)/(c - u)(c + v)]^{1/2} \quad . \tag{121}$$

If the relativistic relative velocity V of the moving points A(u) and B(v) is introduced, Eq. (121) assumes the form

$$T_{Bm}/T_{Am}^* = [(c + V)/(c - V)]^{1/2} \tag{122}$$

where

$$V = (u - v)/(1 - uv/c^2) \quad . \tag{123}$$

Since the measured Doppler frequencies are defined by $\nu_{Am}^* = 1/T_{Am}^*$ and $\nu_{Bm} = 1/T_{Bm}$, Eq. (122) formally agrees with the Doppler effect of the special relativity theory [Whittaker, 1951; Stratton, 1941]

$$\nu_{Am}^* = \nu_{Bm}[(c + V)/(c - V)]^{1/2} \quad . \tag{124}$$

Thus, $\nu^*_{Am} \gtrless \nu_{Bm}$ for $V \gtrless 0$, depending on whether the emitter A moves towards (u > v) or away from (u < v) the moving receiver B. If A and B have the same velocity, no Doppler effect occurs, $\nu^*_{Am} = \nu_{Bm}$ for u = v or V = 0.

It is remarkable that the absolute velocities u of A and v of B relative to the ether frame Σ_o cannot be determined by Doppler measurements. The measured frequency ratio ν^*_{Am}/ν_{Bm} gives via Eq. (124) only the relative relativistic velocity V, since Eq. (123) provides a manifold of solutions (u,v) for any measured value V.

As another illustration of the Galilean relativity physics, it is recalled that the Galilean ideas for anisotropic light propagation parallel ($\parallel$) and perpendicular ($\perp$) to the ether velocity $\vec{w}$ yield for the fringe shift $\Delta \propto \Delta t$ of the Michelson-Morley [1887] experiment.

$$\Delta t = (2/c)[L^{\parallel}(1 - w^2c^2)^{-1} - L^{\perp}(1 - w^2/c^2)^{-1/2}] \quad . \tag{125}$$

According to Eqs. (111) and (112), the phase difference vanishes exactly since the mirror distance $L^{\parallel}$ is Lorentz contracted whereas the mirror distance $L^{\perp}$ is not Lorentz contracted,

$$\Delta t = 0 \text{ for } L^{\parallel} = L_o(1 - w^2/c^2)^{1/2} \ , \quad L^{\perp} = L_o \ . \tag{126}$$

The experimental result $\Delta t = 0$ lead Lorentz via Eq. (125) to the discovery of the physical length contraction of material bodies moving relative to the ether with a velocity $-\vec{w}$.

Thus, it is recognized that the Galilean concepts of space and time, extended with the help of the length contraction of Lorentz and the time dilatation of Builder, explain the Michelson-Morley type interferometer and also Doppler effect measurements, i.e., even their most recent and highly accurate versions [Jaseja et al., 1964; Vessot and Levine, 1979].

The Michelson-Morley maser interference experiment of Jaseja et al [1964] agrees with Eq. (126) up to terms of order $10^{-3}(v_o/c)^2$ where v_o is the orbital velocity of the earth relative to the ether. The maser Doppler measurements of Vessot et al [1972] are in excellent accord with Eq. (124) since these permit to infer an anisotropy of light propagation which is only of the order $\Delta c/c < 10^{-8}$. These investigators believe that the observed, small effects are explainable by means of the general theory of relativity. The latter possibility cannot be discussed here.

CONCLUSION

The original Maxwell's equations for the ether rest frame are generalized to electromagnetic field equations for arbitrary inertial frames, in which the ether is in a state of motion, $\vec{w} \neq \vec{0}$. In connection with this theory, we arrive at the following deductions.

The electromagnetic field equations with ether flow $\vec{w}$ are Galilei covariant, and reduce to the Maxwell equations in the limit $\vec{w}/c \to \vec{0}$. Thus, Maxwell's conjecture that his equations hold in a frame of reference with resting ether is reconfirmed. Maxwell's equations are not Galilei covariant, since they do not refer to an arbitrary inertial frame with moving ether, $\vec{w} \neq \vec{0}$.

Electromagnetic signals propagate isotropically with the speed of light $c = (\mu\varepsilon)^{-1/2}$ relative to the (moving or resting) ether. Relative to inertial frames with ether flow $\vec{w}$, electromagnetic signals propagate anisotropically. The signal velocity is independent of the dynamics of the emitting source, which is typical for wave propagation in a carrier (ether).

In inertial frames with ether flow, the dispersion $\omega = \omega(\vec{k})$ of electromagnetic waves is changed, i.e., an electromagnetic wave of wave vector $\vec{k}$ experiences a blue - or red - shift $\Delta\omega = \vec{k}\cdot\vec{w} \gtrless 0$ for propagation with a wave vector component $\vec{k}_{\parallel}$ parallel (>) or antiparallel (<) to the ether velocity $\vec{w}$. In electromagnetic wave phenomena, the ratio of electric and magnetic field strengths $|\vec{E}|/|\vec{B}| = c + w_{\parallel} \gtrless c$, is increased by the ether flow depending on whether $w_{\parallel} = \vec{k}\cdot\vec{w}/|\vec{k}| \gtrless 0$.

The electromagnetic ether has been incorrectly linked with an "absolute rest system" since Maxwell [Whittaker, 1909]. The substratum is a physical concept [Ives, 1952; Kaempffer, 1953] whereas absolute rest is a methaphysical concept, and these should, therefore, not be intermixed. Within the presented theory, we have only assumed that the ether has different drift velocities $\vec{w}$

in different inertial frames, and that the Galilean relativity principle is valid [Mansouri and Sexl, 1977].

The electromagnetic field equations (14) - (19) with substratum flow $\vec{w}$ and their Galilei transformations represent a field theory in terms of absolute space $\vec{r} = (x,y,z)$ and time t coordinates. The transformations (111) - (112) indicate how these are related to the measured space and time coordinates as a result of length contraction and time dilatation determined by the Lorentz measure $\gamma = (1 - \vec{w}^2/c^2)^{1/2}$. As shown, our theory is in agreement with the Michelson-Morley and Doppler effect experiments.

The infamous difficulties of the Lorentz covariant and ether-free special theory of relativity (applicability to point particles only, violation of causality principle for extended particles, infinite self-energy and self-acceleration of electrons, infinite zero-point energy density of vacuum, twin paradox, etc) are removed by adopting a covariance principle combatible with an electromagnetic ether. The abandonment of Lorentz covariance permits the existence of extended particles and, thus, eleminates infinite self-energy and self-acceleration of the relativistic point particle. The introduction of the ether and a minimum length in quantum electrodynamics gives a finite zero-point energy density of the vacuum [Winterberg, 1984]. Only that twin, who moves relative to the ether, experiences an increased life time, etc.

In addition, the presented theory justufies the widespread use of non-relativistic electrodynamics for the analysis of moving conductors in electrical engineering, magnetohydrodynamics, and plasma physics [Stratton, 1941; Wilhelm, 1983,1984]. We have considered here only the transformation of Maxwell's equations to inertial frames. The theoretical foundations for transforming Maxwell's equations to accelerated reference frames have been laid by Kottler

[1922 a,b], Cartan [1924], van Dantzig [1934], Shouten and Hantjes [1934], and in particular Post [1962, 1967, 1972, 1979]. Electromagnetic ether effects in rotating systems are important for Sagnac interferometers and ring lasers, electromagnetic sensing of absolute rotation, and Sagnac type gyroscopes for navigation, as will be shown in further communications.

AD-A172 071 EXOATMOSPHERIC APPLICATIONS OF OBSCURANTS AND SMOKES 2/2
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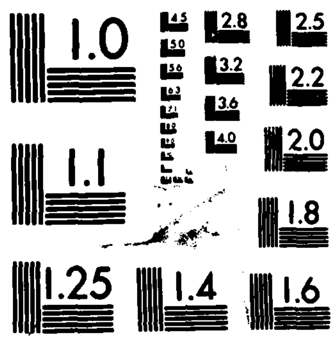

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDA; DS-1963-A

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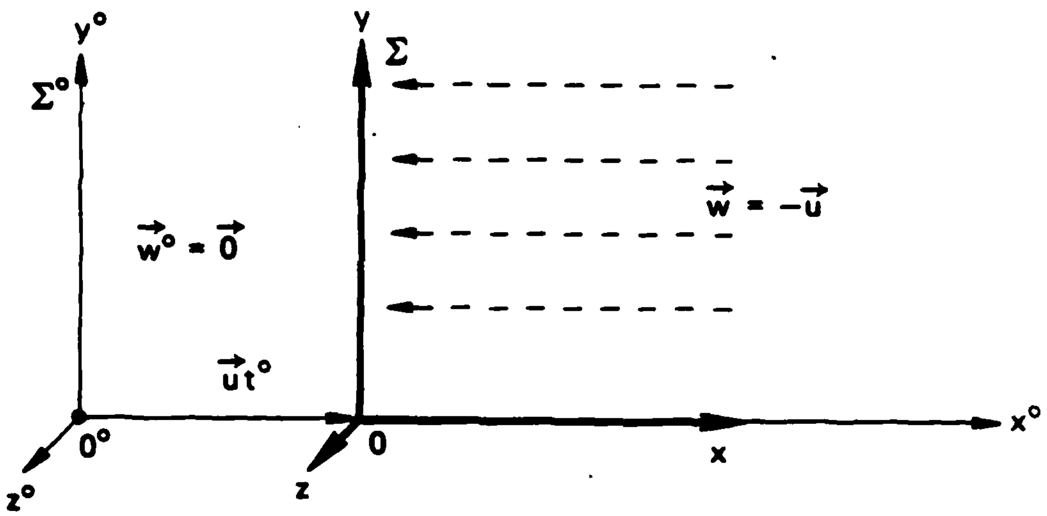
CAPTIONS

Fig. 1: Galilei transformation from inertial frame $\Sigma^\circ(\vec{r}^\circ, t^\circ, \vec{w}^\circ = \vec{0})$ with resting substratum to an inertial frame $\Sigma(\vec{r}, t, \vec{w})$ with streaming substratum $\vec{w} = -\vec{u}$, where Σ moves relative to Σ° with velocity $\vec{u}$ ($0^\circ = 0$ for $t^\circ = t = 0$).

Fig. 2: Galilei transformation from inertial frame $\Sigma(\vec{r}, t, \vec{w})$ to inertial frame $\Sigma'(\vec{r}', t', \vec{w}')$, where Σ' moves relative to Σ with velocity $\vec{u}$ ($0 = 0'$ for $t = t' = 0$).

Fig. 3: Magnetic field pulse $B(x,t)$ (in substratum flow $\vec{w}$) with wave front at $x = (w_{\parallel} + c)t$ produced by switching on a current $\vec{j}^*(t) = \vec{J}H(t)$ on the surface $x = 0$ of a conductor σ.

Fig. 4: Magnetic field pulse $\tilde{B}(x,t)$ (in substratum flow $\vec{w}$) with wave front at $x = (w_{\parallel} + c)t$ produced by the motion $\vec{v}(t) = \vec{v}_0 H(t)$ of a conducting piston σ transverse to a magnetic field $\vec{B}_0$ into the space $x > 0$.



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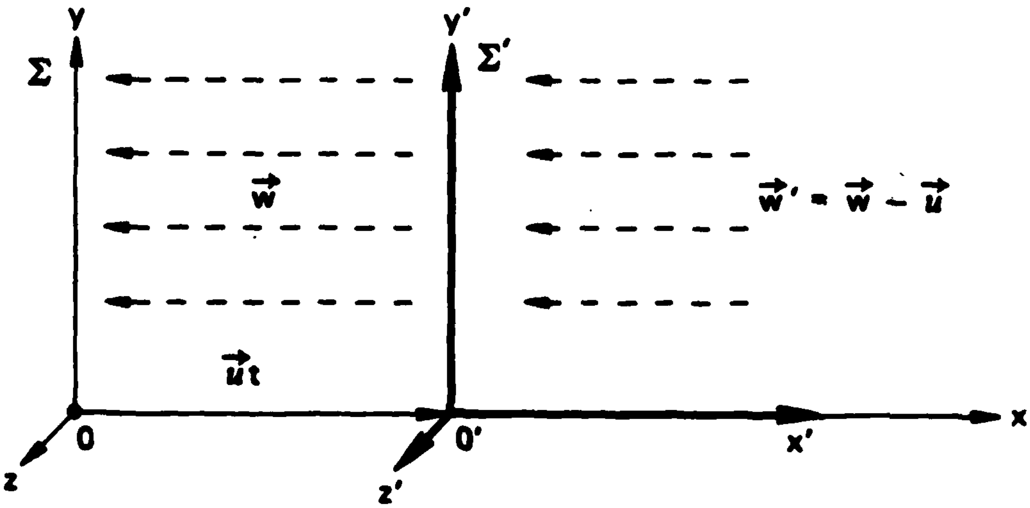

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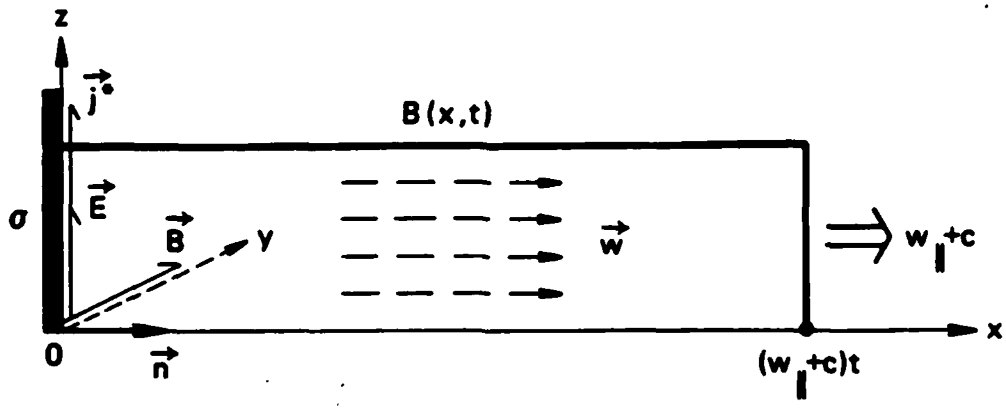

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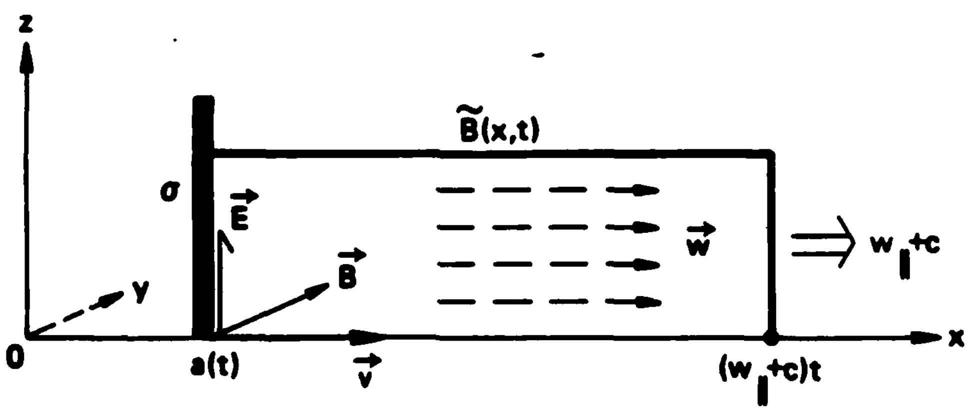

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